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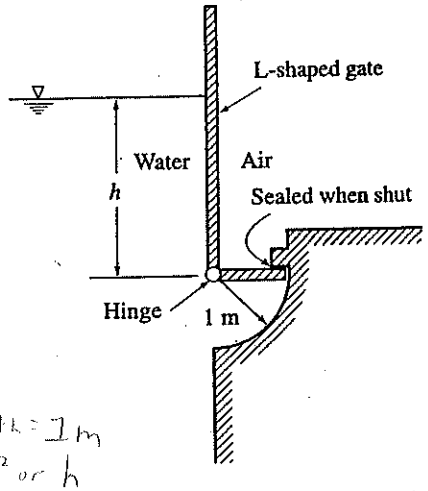
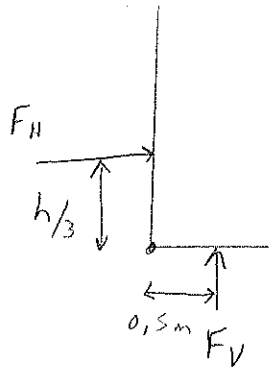
3. (30 pts) An L-shaped, rectangular gate shown below can rotate about the hinge. As the water level rises, the gate will open when the level reaches a critical height, h_c. If the length of the lower horizontal arm is 1 meter, find the critical height, h_c. (Neglect the weight of the gate itself).





Let width = 1m

Area = $1m^2$ or h

$$F_H = \left(\frac{h}{2}\right)\gamma\,(h)(1m) = \frac{h^2}{2}\gamma$$

$$F_V = \gamma h$$

$$\Sigma M_N = 0$$

$$\frac{h^2}{2}\gamma\,\frac{h}{3} = 0.5\,\gamma h$$

$$\frac{h^3}{6} = 0.5h$$

$$h^2 = 3$$

$$h = \sqrt{3} = 1.73\,m$$

$$\boxed{h = 1.73\,m}$$

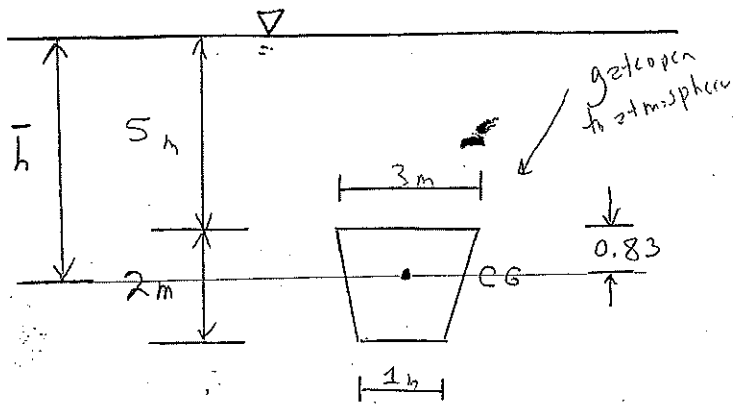
(NON-RECTANGULAR) Vertical, Gate (submerged)

Example 2.3

(pg. 30 HWANG)

A vertical trapezoidal gate w/ its upper edge located 5m below the free surface of water.

Determine the total pressure force & the center of pressure on the gate.



$\bar{y}$ = $\bar{h}$ = below Vertical

Centroid = $\bar{y} = 2 - \frac{h(2a+b)}{3(a+b)}$ from top of object

$= 2 - \frac{2(6+1)}{3(4)} =$ **0.83 m** ↓

Area $= \frac{h(a+b)}{2} = \frac{(2)(1+3)}{2}$

Area = 4 m²

Moment of Inertia (x-axis)

$$I = \frac{h^3(a^2 + 4ab + b^2)}{36(a+b)}$$

$$I = \frac{2^3(3^2 + (4)(3)(1) + 1^2)}{36(1+3)}$$

$I = 1.22\ m^4$

$F = \gamma \bar{h} A = (9810\ N/m^3)(5 + .83\ m)(4\ m^2)$ (ht @ centroid)

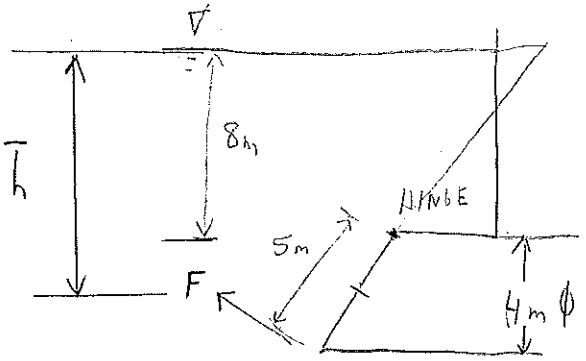
$F = 228{,}900\ N$

Location of C.P.: $y_{cp} = \frac{I}{A\bar{y}} + \bar{y} = \frac{1.22\ m^4}{4 \times 5.83} + 5.83 = 5.88\ m$

Force = 228,900 N
Acting 5.88 m below the surface

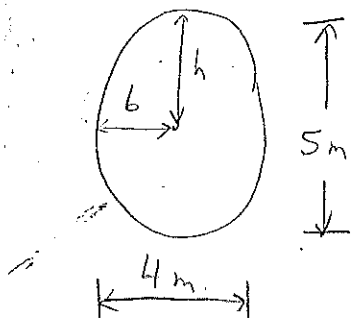
An elliptical gate covers the end of a pipe 4m in ϕ
If gate is hinged @ the top, what normal force, F is required to open the gate when water is 8m deep above the top of the pipe & the pipe is open to the atmosphere on the other side?

Neglect gate wt.



plane surface

(Looked in another book for ellipse)



normal to force

$Area = \pi b h = (\pi)(2)(2.5)$

$Area = 15.7\ m^2$

$Centroid\ (\bar{y}) = 2.5\ m$

$I_{\bar{x}} = \left(\frac{\pi}{4}\right) b h^3$

$= \left(\frac{\pi}{4}\right)(2)(2.5)^3$

$I_{\bar{x}} = 24.5\ m^4$

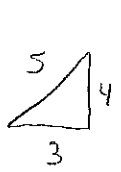
Force Magnitude = $\gamma \bar{h} A$

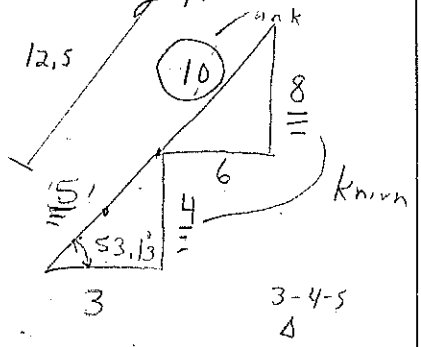
$|F| = (9.81\ kN/m^2)(10\ m)(15.7\ m^2)$

$|F| = 1{,}540\ kN$

LOCATION OF FORCE (Remember $\bar{y}$ is along plane surface.)

$$Y_p = \frac{I_{\bar{y}}}{A\bar{y}} + \bar{y}$$



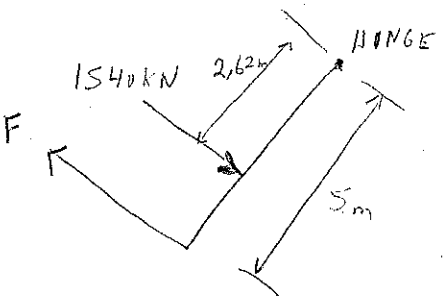


$\bar{y} = 12.5\ m$

$$Y_{cp} = \frac{24.5\ m^4}{(15.7\ m^2)(12.5\ m)} + 12.5\ m$$

$Y_p = 12.62\ m$

$\sin 53.1° = \frac{8}{y'}$



$\Sigma M_H \quad (F)(5m) = (1540\ kN)(2.62\ m)$

$\boxed{F = 807\ kN}$

force required to open gate