CTC/MTC 475-Supplemental Project Manual

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SPREADSHEET MODELING
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CTC/ITC/MTC 475-Supplemental Project Manual

- Breakeven Analyses
- Simple vs. Compound Interest
- Factor Tables
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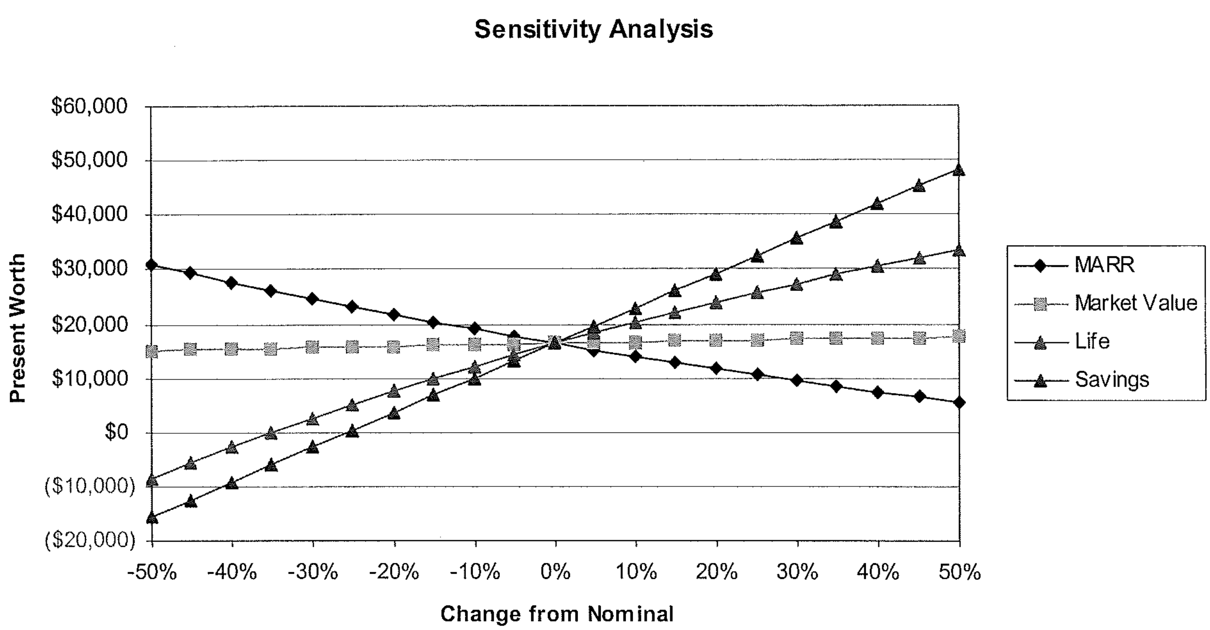


Chapter 1
Spreadsheet Modeling Basics

Introduction

Most engineering economy problems are amenable to spreadsheet solution since:

a) They consist of structured, repetitive calculations that can be expressed as formulas that rely on a few functional relationships.
b) Problems are similar, but not identical.
c) The parameters of the problem are subject to change.
d) The results and the underlying calculations must be documented.
e) Graphical output is often required, as well as control over the format of the graphs.
f) The user desires control of the format and appearance of the output.

Spreadsheets allow the user to develop an application rapidly without getting inundated with the housekeeping details of programming languages. They relieve the user of the drudgery of number crunching, yet still focus on problem formulation. Because of the inherent structure of the spreadsheet, problem formulation is often more thorough than the pencil and paper approach. As a result, misconceptions by the user are often easier to detect. Finally, the flexible nature of the spreadsheet makes it easy to correct the inevitable mistakes that occur during the learning process.

Electronic spreadsheets are put into perspective by comparing them with programming languages commonly used in the engineering curriculum. This chapter covers spreadsheet basics and a summary of spreadsheet commands organized by task for both DOS and Windows.

Differences Between Spreadsheet and Hand Solutions

The solution approach illustrated in the Tenth Edition of Engineering Economy (DeGarmo,. Sullivan, Bontadelh, and Wicks) typically uses closed-form time value of money (i.e., equivalence) relationships. This approach removes the analyst one or more steps from the actual cash flows and may conceal some information that may aid in comprehension of the basic course material. The answer will be the same with either approach, subject to rounding errors, with the spreadsheet being more accurate than the hand solution.

With a spreadsheet, the primary focus is on the actual cash flows instead of the discount factors. The general approach to solving a problem with a spreadsheet consists of several basic steps:

1) Plan the approach to the problem before sitting at the keyboard. Cash flow diagrams and/or tables are useful.
2) Enter the periodic cash flows, the interest rate (MARR) and any other pertinent information from the problem statement into separate cells of the spreadsheet.
3) Determine and enter the necessary formulas (generally needed only for time period 1).
4) Copy the formulas as required over all other time periods to complete the model.
5) Use the appropriate formulas and/or functions to arrive at a measure of merit (PW, IRR, etc.).

6) Verify the formulas and/or functions by using sample values (O and 1 work well). Values from solved examples also can be used.
7) Document and save the spreadsheet template for future use.

Getting Started

The templates in this document use *Excel 97,* since it currently enjoys widespread usage in industry and academia. Although *Excel 97* is illustrated, the templates are compatible with earlier releases,. except for some formatting options, newer functions, and the Solver command.

Other excellent spreadsheet software, such as Excel and *Quattro Pro,* among others, can be used with equal facility. Since most spreadsheets currently on the market behave similarly to Excel, the general procedures outlined here are readily applicable to other software packages. Keyboard commands are used throughout, although the reader may want to become familiar with the shortcut commands for his or her specific software package.

Readers should be familiar with their hardware and the basic features of its operating system, such as formatting disks, copying files, and loading specific spreadsheet packages. Although the functions used in this presentation are common to almost every spreadsheet package, you should verify the availability and syntax for your specific software. Commands for graphing and the Solver exhibit the most variation between spreadsheet packages.

This manuscript emphasizes the development of flexible models instead of an exact keystroke-by-keystroke presentation. Templates illustrate the basic procedures required to solve nearly every type of economic analysis problem presented in the Tenth Edition of *Engineering* Economy. Cells that contain user input and unique formulas are identified on each template. Usually, cells to the right or below unique cells are obtained with the copy command.

Spreadsheet Basics

Most engineering students have been exposed to programming languages by the time they take this course. Since the same is generally not true about spreadsheets, we briefly compare and contrast features of both for the benefit of new users. A screen display of a typical spreadsheet is shown in Figure 1- 1.

Spreadsheets are highly interactive, requiring no linking or compiling as changes are made to either the model or the data. Results are updated in real time as values or formulas are changed. Detailed knowledge of operating system command is not required, since many tasks, such as file handling, are available internally.

The spreadsheet has few structural constraints. Results are unaffected by location within the model, so input, results, and text can go wherever needed to provide clarity.

Iterative structures, such as *for-next* or *do* loops, are available with macro commands, but this approach is typically not used. Instead, a column of numbers corresponding to the values that will be assumed by the loop index variable, is created using either a menu command or formula. A formula with the desired relationship is entered adjacent to the first value in this column and is copied over a parallel range. The resulting two columns contain the values of the index and the corresponding value of the formula for that value. This procedure is illustrated in subsequent templates.

Tasks normally accomplished with subroutines, such as sorting and basic statistical functions, are done with macros, functions, or menu commands. The macro language has many features common to typical programming languages. To ensure the greatest compatibility with other software, and even earlier versions of Excel, macros are not used in any of the templates in this manuscript.

Viewing the spreadsheet as a 2-dimensional matrix referenced in column order is helpful for novice users. Each element of the matrix can be considered a (potential) variable, whose default name is its location in the matrix. The elements of the matrix correspond to the *cells* of the spreadsheet.

The cell is the basic building block of the spreadsheet and is identified by its column letter and row number. In Figure 1-1, the label **"SAMPLE SPREADSHEET'** is in cell C2, the intersection of column C and row 2. Note that the display spills into cell D2. Cell Al is in the upper left corner and is called the *home* cell.

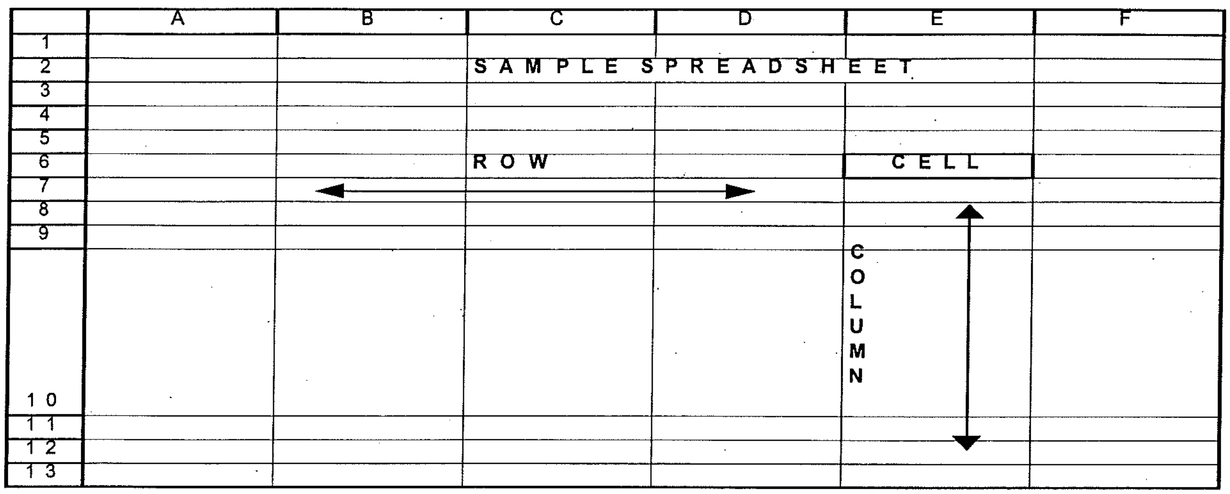


Figure 1-1. Sample screen showing the basic spreadsheet components.

Programming languages assign values to variables with an assignment statement, such as X =7 + 49 **a.* In a spreadsheet, the cell location is used as the (default) variable name. Assigning a value to a cell is done by moving the cell pointer to the desired cell and entering the appropriate information. Formulas, values, and labels can be entered in any cell, and can reference the contents of any other cell. The default display format shows the formula results, not the actual formula. The formula may be viewed on the command line by placing the cell pointer on that cell.

The other fundamental unit is the range, which can be a single cell, a portion of a row or column, or any uniform rectangular region. A range is identified by its first and last cells with column between them (e.g., A1:A15). A one-cell range has the same starting and ending cell (e.g., B1:B1), while a rectangular range uses the top left and bottom right cells (e.g., A1:G7). When prompted for a range by a worksheet command, move the cell pointer to the first cell in the range and anchor the range by pressing the period key. Then move to the last cell in the range and press the "Enter" key.

There are four major ways to make things happen in a spreadsheet model: User Input, Formulas, Function Keys, and Menu Commands. User Input consists of parameter values, any explanatory text, and row and column headings. Formulas contain the fundamental engineering economy relationships to model the problem. Function Keys and Menu Commands build and enhance the model. Menu Commands also perform such functions as printing, graphing, and file access.

Figure 1-1 illustrates the *Excel 97* screen. Many Windows-based spreadsheets have similar structures. The upper five lines form the control panel. The title bar appears on the top line and identifies the software and current file name. The second line contains the main menu. These items are the top level menus for all menu commands. The fifth line is the edit line, which displays the address and contents of the active cell. In this figure, C2 is the active cell and it contains the label "SAMPLE SPREADSHEET."

The third and fourth lines contains *Excel* default Smart Icons, which are shortcuts for frequently performed tasks, such as opening and closing files and formatting cells. Similar features are available, by different names and icons, in other spreadsheets.

The next line starts the actual worksheet window. The tab (A1) at the top shows that we are on the first worksheet and no others are present. With the exception of one template, all spreadsheets in this manuscript use only one worksheet.

The actual worksheet is bounded by the row indicator on the left and the column indicator on the top. It is in this area that all input takes place and where all results are displayed. Other portions of the worksheet are exposed with the pointer-movement keys or mouse.

The status bar is the last line in the window. It contains useful information and several shortcuts. The first block, which contains "Ready", indicates the format for the current cell.

Formulas

A generic spreadsheet model (template) for a class of problems is created by entering formulas that contain the key relationships. Formulas may include functions. The model is made specific for the problem at hand by entering values into cells (User Input) that are referenced by the formulas to arrive at the answer for the current problem. Every time these input values are changed, all formulas are automatically updated. Labels identifying input cells and row and column headings make it easier to interpret and revise the model.

Cell Pointer Movement

Input is accomplished by moving the cell pointer to the desired cell and entering information- While many users rely on the mouse to move the cell pointer, the keyboard commands are frequently faster. Keyboard movement is done one cell at a time with the arrow keys (-, -, T, 1). Moves of a full screen are accomplished with PGUP, PGDN, TAB, and by holding SE= and pressing TAB. The HOME key will return to cell Al from anywhere in the spreadsheet. Pressing the END key followed by an arrow key will move over blank cells to the first nonblank cell or move to the last filled cell in the direction specified by the arrow key.

Types of Input

The spreadsheet must be in the "Ready" mode (refer to the status bar) to accept input. If you are not in the Ready mode, you are probably within the hierarchial menu system. Pressing the "ESC' key will move you up one menu level at a time until you eventually return to the Ready mode.

A cell may contain numbers (e.g., -1.23, 123456789, 49E-15), formulas (e.g., =A23*5, =7+4*2.3, =5+min (A1: A10), or labels (e.g., Alternative Alpha). The software determines the type of input by the first character in the cell. Cells can be edited by moving to the desired cell and double clicking the mouse or pressing the F2 key. The arrow keys will now move the cursor within the expression. Use the "Delete" or "Backspace" key to delete characters. The "Insert" key toggles between insert and type over mode. Spreadsheets are not case sensitive.

Documenting the values and formulas in the spreadsheet with explanatory labels is helpful. Labels are left justified by default. Labels longer than the column will spill over into successive blank columns, as shown in cell D2 in Figure 1-1.

Numbers are right justified by default and are entered without commas or currency signs.

These features are added by changing the format of the cell.

Formulas consist of algebraic expressions and functions. Reference to values in other cells is done by either (1) pointing to the cell with the arrow keys, (2) typing the cell address directly, or (3) entering the range name, if one has been assigned. If the first character in a formula is a reference to another cell, it must be preceded by a "+" sign (or "-" sign if appropriate) or it will be treated as a label (since the first character is the column designation), and the expression will not be evaluated. Expressions are evaluated left to right and follow typical precedence rules. Exponentiation is denoted by "^", although some software packages use

Financial Function Summary

The financial functions are based on the following assumptions, which agree with those presented in the Tenth Edition of *Engineering Economy*:

(a) The per period discount rate, *i*, remains constants
(b) There is exactly one period between cash flows.
(c) The period length remains constant.
(d) The end of period cash flow convention is used.
(e) The first cash flow in a range occurs at the end of the first period.

The last assumption needs to be emphasized, since most problems involve an investment at time k = 0, which is the beginning of the first period. This only affects the PW calculation. Be sure to review the assumptions made in your particular software.

The most useful financial functions are:

PV(A,i,n)	Returns the present worth of an annuity.
FV(A,i,n)	Returns the future worth of an annuity.
PNIT(P,i,n)	Returns an annuity given a present worth.
NPV(L', range)	Returns the net present worth of any cash flow.
IRR(guess,range)	Returns the internal rate of return for a cash flow range.

Note that there is no space preceding the left parenthesis. A, i, and N, are single values and have the same definition as in Chapter3 of the Tenth Edition of *Engineering Economy.* Range is the cash flow range and *guess* is a decimal estimate of the IRR needed to begin the IRR solution process. The equivalent worth functions return dollar amounts, not the discount factor values. Other functions, including depreciation, are available. Check your particular spreadsheet for their availability. Additional functions are introduced as needed in the templates.

Function Keys

The function keys (Fl - FIO on the keyboard) provide some unique features and shortcuts for some spreadsheet commands. The most useful keys are:

F 1: Context sensitive help. Especially useful for function arguments.

F2:	Edit the contents of a cell.
F4:	Used to select type of addressing when typing cell names or pointing to cells. Each time the key is pressed, it cycles through absolute, mixed row, mixed column and relative.
F9:	Recalculate the spreadsheet.
ESC:	Although not a function key, it is useful to back out of commands, leave help, and leave the edit mode.
ENTER:	Completes the current editing action, accepts an option, or selects a command.

Menu Commands

All other aspects of model formulation, appearance and output are handled by menu commands. The top level menu is always present in the second line of the control panel. Selections are made by highlighting the desired command. This can be done with the mouse, or by pressing the "Alt' key and then typing the underlined letter. Most menus contain submenus, which are accessed in the same way. To exit a menu, press the "Esc" key.

Spreadsheet Appearance

The appearance of the spreadsheet can be enhanced for clarity. While the display will change, the underlying formulas and values are preserved.

Blank rows or columns can be inserted with Edit Insert and then completing the dialog box. Rows and columns can be deleted with the Edit Delete dialog box. Be careful when deleting rows and columns: remember rows are horizontal, columns are vertical. Larger areas of the spreadsheet can be cleared by highlighting the range and using the Edit -Clear command. The dialog box allows one to delete cell contents and/or cell styles. If information is accidentally deleted, it can be recovered using the Edit -Undo command.

The appearance of numbers, such as the number of significant digits and the presence of dollar and percent signs or commas, is changed with the Style Number Format command or by clicking on the status bar. Figure 1-2 illustrates the results of several common formatting and alignment options. Formulas can be displayed as text, or the numerical result of the formula (default).

Long labels with nonblank cells to the right and large numbers may require wider columns to display their results. This can be done for each column individually using the Style Column Width command or by moving the cursor to the edge of the column indicator and dragging in the appropriate direction.

Labels shorter than the column width may be aligned left ('), center (') or right (") by using the corresponding prefix as the first character, see the upper right portion of Figure 1-2. Alternatively, labels can be aligned with Style Alignment command.

Moving Cells

Cell contents are removed by highlighting the range and then using the Edit Cut command. Cell contents are placed in a new location by highlighting the first cell in the new range and using' the Edit Paste command. Anything in the new range will be replaced with the new material.

The original location will be blank after the Cut command is issued. When a formula is cut and pasted in a new location, the cell references of the formula are automatically updated to reflect its new location. If a value is cut and pasted to a new location, all formulas that refer to it are automatically revised to reflect the new location.

Copying Cells

To copy the contents of a cell to a new location, while leaving the original cell intact, highlight the cell or range and use the Edit -Copy command. Move the cursor to the new location and use the Edit Easte command. Formulas in copied cells are modified, as described below.

Cell Addressing

The manner in which cells in formulas are referenced internally provides the power and flexibility of the spreadsheet. These features make it possible to construct complex models using a minimal number of formulas and to get updated results as values are changed. Many problems encountered by novice spreadsheet users can be traced back to a lack of understanding of how cells are referenced.

There are three methods to refer to the address of a cell: relative, absolute and mixed. The distinction between them becomes important only when copying formulas. Assume that cell B5 contains the formula +A4+1 and cell A4 contains the value 7. The result displayed in B5 is 8.

	A	B	C	D	E	F
1						
2	**Number Format**			**Label Alignment**		
3	general:	123456		Left		
4	fixed (2):	123456.00		Right		
5	sci (2):	1.23E+05		Center		
6	currency (0):	$123,456		repeating repeating repeating		
7	comma (1):	123,456.00				
8						
9	**Number Alignment**					
10	1			**Long Label That Exceeds Column Width**		
11	2			**Long Label That Exceeds Col**	*New Label*	
12	3					
13						
14						
15	**Displaying Formulas**					
16	Formula (default display):			6		
17	Formula (text format):			=SUM(A10..A12)		
18						
19	Figure 1-2 Sample screen illustrating various format and alignment options.					
20						

Although the formula refers to cell A4, internally the spreadsheet interprets it as: "add 1 to the contents of the cell that is one row up and one column to the left of this cell." This is relative addressing, which is the default. When the formula in B5 is copied, the cell that receives the copy of the formula will display the sum of 1 plus the contents of the cell that is one row up and one column to the left of the new cell.

There are occasions when one always wants to refer to a specific cell, no matter where a formula is copied. The discount rate, i, is typically a value that is constant for a problem. Absolute addressing (a $ in front of both the column and row designation, e.g., A 1) will always return the contents of that specific cell, no matter where the formula is copied.

Sometimes it is necessary to hold either the row or the column constant and allow the other to vary. This is achieved with mixed addressing. A$ is placed in front of the row or column that is to be kept constant. Thus, $Al will make the column reference absolute and the row reference relative, while A$l will make the row reference absolute and the column reference relative.

Figure 1-3 illustrates the results of the cut, copy, and paste commands for the three types of addressing. The range A5..C9 contains arbitrary values. Cells E5..E9 contain formulas that compute the respective row totals. The actual formulas are shown in column F. Results of cut, copy, and paste actions are shown in range E 13..E 17, and the resulting formulas are in column F.

				Original Formulas			
Data Values			Result	Underlying Formula			
1	1		3	SUM(A5..C5)			
2	2		6	SUM(A6..C6)			
3	3		9	SUM($A$7..$C$7)			
4	4		12	SUM(A$8..C$8)			
5	5		15	SUM($A9..$C9)			
	Action		Result	Underlying Formula			
	Cut E5, Paste in E14		3	SUM(A5..C5)			
	Copy E6 to E15		0	SUM(A15..C15)			
	Copy E7 to E16		9	SUM($A$7..$C$7)			
	Copy E8 to E17		12	SUM(A$8..C$8)			
	Copy E9 to E18		0	SUM(A18..C18)			
Sample screen illustrating effect of cut and paste operations on formulas							

Figure 1-3. Sample screen illustrating effect of cut and paste operations on formulas.

Printing

Once the spreadsheet model is completed, it is usually printed for presentation to others and for documentation. Printing is done with the File Print command. Use the File Print Preview command to examine the worksheet on the monitor before printing to save time and paper. For additional control over the final format, use the File Page Setup command. Options in this dialog box allow you to add headers and footers, center the output, select page orientation, and include grid lines and the worksheet file name on the printed page. Unless a range or chart is highlighted, the default is to print the entire worksheet.

Graphing

Graphs are created after the spreadsheet is complete. See the Tools Chart command. Most recent software releases provide a series of dialog boxes to help in the chart creation process.

File Services

Each spreadsheet is saved as a separate file with the File Save command. Use the File Save As command to save a template under a different name. It is good practice to do this before making any changes to a template, since the automatic backup feature on many software packages may result in unintentional modification of the original template.

Previously created files are retrieved with the command File Open command. Most current software will open the new file while retaining the current file in memory. Use the Window command to move between open files.

Structured Spreadsheets

Using a structured approach for model formulation reduces the chance of overlooking information, aids in locating errors and making revisions, and makes it easier for others to interpret your analysis. Taking a few minutes to organize the data and plan the model before sitting at the keyboard will reduce the total development time. There is no single "best" format, but the following tips work well in practice.

a. Treat everything as a variable, even if you do not expect it to change. Assign key parameters, such as the discount rate, life, and estimates of revenue and expenses, their own cells. Formulas should reference these cells, not the values themselves. This way, if these values change, it is necessary to revise these few cells, rather than try to locate all of the formulas that contain these values. If these cells are in the upper left corner of the spreadsheet, one can return to them from anywhere in the spreadsheet by pressing the home key.

b. Next, divide the problem into segments. Combining several calculations into a single formula saves space, but such an approach makes it more difficult to verify results and locate errors. Each segment should provide an intermediate result, which can be referenced in other segments and aids in isolating problems. This recommendation also applies for different types of cash flows. Rather than combining revenues and expenses for each period before entry on the spreadsheet, list each type of cash flow in a column and sum them to get a net figure for each period.

c. It is helpful to save each type of problem as a separate file, which is called a template. When a similar problem is encountered in the future, the appropriate template is retrieved, revised, and a new analysis is completed. If you are creating a new spreadsheet from a template, be sure to change the file name the first time you save it, or the template will be overwritten.

d. It is possible to create new functions, automate commands and create new menus by using *macros*. These are beneficial if you will be using a template frequently, but macros are generally not productive in the classroom setting.

e. *Range* names are useful when entering long formulas and when using macros. They can be assigned to any valid range (including a single cell) with the Range Name command. Range names can be made absolute by prefixing the name with a dollar sign. In some spreadsheet packages the range name must be defined before it can be referenced in a formula.

Using This Document

The remaining chapters of this document contain templates that illustrate useful menu commands and functions, in addition to basic model formulation techniques needed to solve engineering economy problems. With these templates and an understanding of fundamental engineering economy relationships, you should be able to easily construct spreadsheet models for all problems in the Tenth Edition of *Engineering Economy*. Keep the following points in mind.

a. Remain familiar with the hand solutions, so the spreadsheet model can be verified.
b. Understand the course material first, have a plan for solving the problem, and then use the spreadsheet to help with the calculations.
c. Try the templates to verify that they are working properly with your particular software and then tackle the end of chapter exercises.
d. Blindly copying formulas and pressing keys without taking the time to look at what is going on does not aid learning. Take time to understand how the template works.
e. Graphing your results can reveal relationships that may not be easily discernable when looking at columns of numbers.

Chapter 2
Cost and Economic Concepts and the Role of Design

Tenth Edition Highlights

Chapter 2 focuses on key economic terms and concepts. While they appear simple, a full understanding of their meaning and application is required to be successful with the material in subsequent chapters. The distinction between cash and book costs, sunk and opportunity costs, and cost classifications are particularly important. The concept of life cycle costing is fundamental to reliable investment decisions.

This chapter presents examples of several types of present economy studies. These studies focus on the cash flow magnitudes, while ignoring both the timing of the cash flows and the earning power of money. Present economy problems are common in industry, taking the form of make vs. buy decisions, selecting between different materials, and specifying different machining speeds. Breakeven analysis is often used to attack these problems.

Overview

Most current spreadsheet software versions offer three approaches for solving problems: (1) General, (2) Goal Seek, and (3) Solver. The general approach will work with any spreadsheet software package, while the more powerful Goal Seek and Solver features are available on newer versions.

No matter which approach selected, one must develop a formula for the measure of merit (e.g., total cost, average cost, etc.) that is a function of the decision variable (e.g., number of units to produce, hours of operation, etc.) for each alternative. The breakeven point occurs when both functions are equal, or equivalently, when the difference between them is zero.

The general approach is a brute force enumeration of the measure of merit values for each alternative over a range of the decision variable. One scans the measure of merit values to determine when they are equal to identify the breakeven point. Template 2-1 shows this approach for linear breakeven analysis, along with some tips to reduce the amount of space required on the spreadsheet. While this approach is not particularly elegant, it will work on any software package and is necessary if one wants to plot the functions.

Most current software versions provide a variety of *solver* tools to simplify problem formulation. Template 2-2 repeats the analysis of Template 2-1 using the Goal Seek menu command.

Spreadsheets can also handle nonlinear breakeven problems. Templates 2.3, 2.4, and 2.5 illustrate the General, Goal Seek approach.

Spreadsheet Templates

2.1	bkevnl.xls	Linear breakeven

2.2	bkevn2.xls	Linear breakeven, Goal Seek menu cornmnnd
2.3	bkevn3.xls	Nonlinear breakeven
2.4	bkevn4.xls	Nonlinear breakeven, Goal Seek menu command
2.5	bkevn5.xls	Nonlinear breakeven, Solver menu command

New Features and @Functions

<u>Logical Operators</u>

=, <, >, <>, <=, >=, #AND#, #OR#, #NOT#

The contents of cells can be compared with constants or other cells using these operations. A 1 is returned if the condition is true, a 0 if the condition is false.

<u>IF(condition, true.false,)</u>

The IF function provides a great deal of power and flexibility for spreadsheet models.

The condition argument is a logical expression. If the result of the condition is true, the value or label in the true argument is returned. If the result is false, the value or label in the false argument is returned.

<u>Goal Seek</u> Range / Analyze / Goal Seek.....

Goal Seek gets its name from its approach to solving the problem. The analyst specifies the desired end result, the cell containing a formula that calculates that result, and a cell referenced in the formula that will be adjusted to achieve the end result.

This tool works best for problems with a single solution, making them a good choice for linear breakeven problems.

<u>Solver</u> Range / Analyze/ Solver...

Solver is appropriate for problems where the analyst does not know what the end result is, only that it should be as large or as small as possible. The analyst specifies a cell containing a formula to optimize, cells that contain values referenced in the formula that can be adjusted, and cells with logical conditions that constrain the solution.

Because of its ability to evaluate multiple solutions, this is an excellent tool for nonlinear breakeven analysis. Various detailed reports on the solution and the solution process are available.

Template 2.1: Linear Breakeven, General Approach

Template 2.1 illustrates linear breakeven analysis between two alternatives based on total cost. The analyst must choose between two processes (Alpha or Beta), each with different fixed and variable costs. Alpha is obviously better suited for very small runs, but Beta is more cost effective for larger runs. Breakeven analysis shows when process Beta is more cost effective than process Alpha.

Spreadsheets are most flexible when all variables appear in a central location and all formulas refer to these cells. As information changes, one simply changes these few cells to update the entire spreadsheet. The alternative names are specified as variables in cells B3 and C3. Cells B4 and C4 contain the fixed setup costs and the variable per piece costs are in cells B5 and C5.

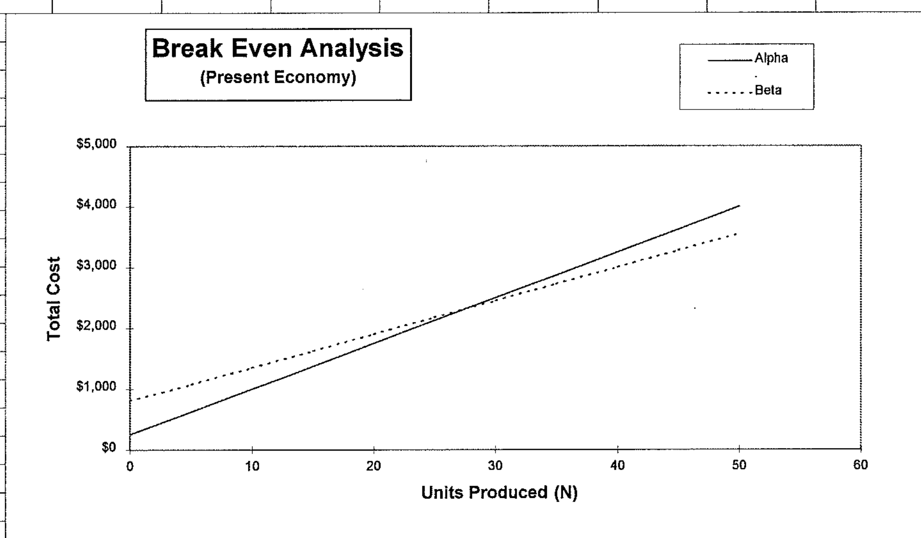
Since we do not know where the breakeven point is, we would have to specify a large range of values for the decision variable, N. Because our interest is only in the breakeven point, we can save space by specifying an arbitrary starting point and increment for *N.* This approach allows us to move to the approximate region of the breakeven point quickly, and then narrow our focus by selecting different constants.

Cell A14 copies the start point to a new location. The formula in cell A15 adds the increment to the cell immediately above it. Absolute addressing is used to refer to the increment value in cell B9. Total cost is calculated forAlpha and Beta in cells Bl4 and Cl4. Again, note the use of absolute addressing to refer to the set up and per piece costs in the total cost formulas. To make it easier to spot the breakeven point, cell D14 returns the name of the lower cost alternative.

To complete the template, the formulas in cells A15, B14..Dl4 are copied down their respective columns. The ranges B14..B24 and C14..C24 are used to create a graph of the total cost relationship. The alternative names are referenced with the formulas in B12 and C12.

Template 2.1 shows that the breakeven point occurs somewhere between 25 and 30 units. To get a more accurate answer, enter the value 25 into cell B8 and the value I into cell B9. All formulas will automatically recalculate to provide a greater level of detail in the region of the breakeven point. The graph will also be redrawn.

	A	B	C	D	E	F	G
1	**Present Economy Linear Breakeven Analysis**					**Template 2.1**	
2							
3	Option:	Alpha	Beta			KEY:	
4	Set-up cost:	$250	$800				= User input
5	Per piece cost:	$75	$55				= Unique formula
6							
7	Units produced					Unique Formulas	
8	Start:	0				B12	=B$3
9	Increment:	5				A14	=B8
10						B14	=B$4+B$5*$A14
11	**Units**	**Total Cost for N Units**		**Least Total**		C14	=C$4+C$5*$A14
12	**Produced (N)**	**Alpha**	**Beta**	**Cost Option**		D14	= IF(B14>C14,$C$3,$B$3)
13						A15	= A14+$B$9
14	0	$250	$800	Alpha			
15	5	$625	$1,075	Alpha			
16	10	$1,000	$1,350	Alpha			
17	15	$1,375	$1,625	Alpha			
18	20	$1,750	$1,900	Alpha			
19	25	$2,125	$2,175	Alpha			
20	30	$2,500	$2,450	Beta			
21	35	$2,875	$2,725	Beta			
22	40	$3,250	$3,000	Beta			
23	45	$3,625	$3,275	Beta			
24	50	$4,000	$3,550	Beta			



Template 2.2: Linear Breakeven, Goal Seek Menu Command

The problem in Template 2.1 is repeated using the Goal Seek menu command instead of general enumeration. Note in Figure 2-1 that the major differences between the templates are the addition of a difference cell (B13) and the elimination of the formulas copied down columns A through D.

The initial Goal Seek dialog box, shown in Figure 2-1, prompts the analyst to specify the cells used to reach a solution. Cell B13 is the difference in total cost between the two alternatives. When this difference is zero, the breakeven point is reached. Goal Seek will adjust cell A11, the number of units produced, until the breakeven point is reached. For linear breakeven, the initial value in the adjustment cell (A11) is not critical.

Figure 2-2 shows the spreadsheet after clicking on the OK button in the Goal Seek dialog box. Note that the breakeven point, 27.5 units, which would be rounded to 28 units, appears in the cell specified as the adjustment cell.

While the Goal Seek solution is more direct and saves space, we are unable to plot the results. Since graphics are important communication tools when selling recommendations to decision-makers, a hybrid approach should be employed. The Goal Seek can easily be used with Template 2.1 to get an exact breakeven point and still provide graphic output.

	A	B	C	D	E	F	G
1	**Present Economy Linear Breakeven Analysis**					**Template 2.2**	
2							
3	Option:	Alpha	Beta				
4	Set-up Cost:	$250	$800				
5	Per Piece Cost:	$75	$55				
6							
7							
8	**Units**	**Total Cost for N Units**					
9	**Produced (N)**	**Alpha**	**Beta**				
10							
11	0	**$250**	**$800**				
12							
13	Difference:	**($550)**					
14	Least Cost Option:	**Alpha**					
15							
16	**KEY:**						
17		=User Input					
18		=Unique Formula					
19							
20							
21	**Unique Formulas**						
22	B9	=B$3					
23	C9	=C$3					
24	B11	=B$4+B$5*$A11					
25	C11	=C$4+C$5*$A11					
26	B13	=B11-C11					
27	B14	=IF(B13,0,B3,+C3)					
28							
29	**Figure 2-1. Template 2-2 Initial Goal Seek Set-up**						
30							

	A	B	C	D	E	F	G
1	**Present Economy Linear Breakeven Analysis**					**Template 2.2**	
2							
3	Option:	Alpha	Beta				
4	Set-up Cost:	$250	$800				
5	Per Piece Cost:	$75	$55				
6							
7							
8	**Units**	**Total Cost for N Units**					
9	**Produced (N)**	**Alpha**	**Beta**				
10							
11	27.5	**$2,313**	**$2,313**				
12							
13	Difference:	**$0**					
14	Least Cost Option:	**Beta**					
15							
16	**KEY:**						
17		=User Input					
18		=Unique Formula					
19							
20							
21	**Unique Formulas**						
22	B9	=B$3					
23	C9	=C$3					
24	B11	=B$4+B$5*$A11					
25	C11	=C$4+C$5*$A11					
26	B13	=B11-C11					
27	B14	=IF(B13,0,B3,+C3)					
28							
29	**Figure 2-2. Template 2-2 Goal Seek Solution**						
30							

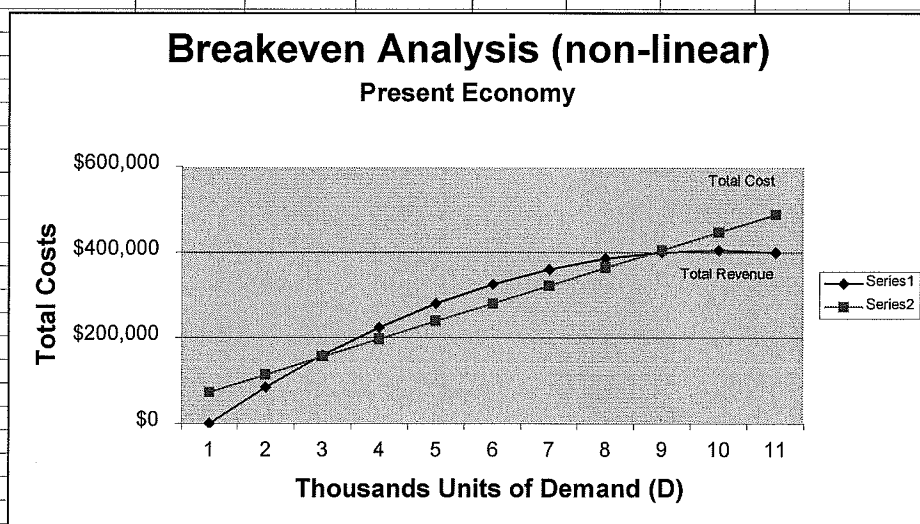
Template 2.3: Nonlinear Breakeven, General Approach

Example 2-6 in the Tenth Edition of *Engineering Economy* (DeGarmo, Sullivan, Bontadelli, and Wicks) uses closed form analysis to maximize profit. The equivalent spreadsheet model is shown in Template 2.3. The revenue and cost constants appear in B4, B5, B8, and B9. Cells B12 and B13 provide a start point and increment for the range of decision variables, as was done in Template 2.1.

The total revenue formula appears in cell B16 and the Total Cost function is in cell C16. The difference between these is profit, which is in cell D16. These formulas are copied down to complete the table. Cells B16..B26 and C16..C26 are plotted.

Note that two breakeven points are present in the graph. These can be examined in greater detail by adjusting the start and increment cells. The maximum value of profit can also be identified using the same approach. Grid lines added to the graph aid in estimating the breakeven points.

	A	B	C	D	E	F	G	H	I	J
1	**Present Economy Non Linear Breakeven Analysis**						**Template 2.3**			
2										
3	**Total Revenue Function Constants**						**KEY:**			
4	a=	$180.00						=User Input		
5	b=	$0.02						=Unique Formula		
6										
7	**Total Cost Function Constants**						**Unique Formulas**			
8	Fixed =	$73,000.00					A16	=B12		
9	Variable =	$83.00					B16	=$B$4*A16-$B$5*A16^2		
10							C16	=$B$8+$B$9*A16		
11	**Demand Range**						D16	=B16-C16		
12	Start =	0					A17	=A16+$B$13		
13	Increment =	500								
14										
15	**Units Demanded (D)**	**Total Revenue**	**Total Costs**	**Profit**						
16	**0**	**$0**	**$73,000**	**($73,000)**						
17	**500**	$85,000	$114,500	($29,500)						
18	1000	$160,000	$156,000	$4,000						
19	1500	$225,000	$197,500	$27,500						
20	2000	$280,000	$239,000	$41,000						
21	2500	$325,000	$280,500	$44,500						
22	3000	$360,000	$322,000	$38,000						
23	3500	$385,000	$363,500	$21,500						
24	4000	$400,000	$405,000	($5,000)						
25	4500	$405,000	$446,500	($41,500)						
26	5000	$400,000	$488,000	($88,000)						
27										
28										



Template 2.4: Nonlinear Breakeven, Solver Menu Command

Template 2.4 is Template 2.3 modified for use with the solver menu command. Note in Figure 2-3 that cell A14 contains the value 0. Solver returns a breakeven point of 931.5 units to this cell.

The graph in template 2.3 shows that there are two solutions to this problem. To get the second solution, enter a value in cell A14 that is closer to the second solution. In Figure 2-4, the value 4,000 is used. Solver now returns a value of 3,918.5 units.

There is no direct method to use Solver to determine the maximum profit, since we do not know *a priori* what that value is. Solver must be used caution, as illustrated in this template. It would be very easy to take the first answer returned and consider the problem solved. The same tool that simplifies the problem also removes some control from the analyst.

Present Economy Non Linear Breakeven Analysis					Template 2.4		
Total Revenue							
Function Constraint							
a =	$180,00			KEY:			
b =	$0,02				User input		
Total Cost Function					Unique Formula		
Constraint				B14	B$5A14-B$6*A14^2		
Fixed =	$73.000,00			C14	B$9+B$10A14		
Variable =	$83,00			D14	B14-C14		
Units Demanded	Total	Totaal Costs	Profit				
(D)	Revenue						
0	0	73000	-73000				

Figure 2-3. Template 2.4 Initial Dialog Box.

Present Economy Non Linear Breakeven Analysis						**Template 2.4**
Total Revenue						
Function Constraint						
a =	$180.00			KEY:		
b =	$0.02				=	User input
Total Cost Function					=	Unique Formula
Constraint				B14		B$5*A14-B$6*A14^2
Fixed =	$73,000.00			C14		B$9+B$10*A14
Variable =	$83.00			D14		B14-C14
Units Demanded	**Total**	**Totaal Costs**	**Profit**			
(D)	**Revenue**					
4000	400000	405000	-5000			

Figure 2-4. Template 2.4 Second solution

Template 2.5: Nonlinear Breakeven, Solver Menu Command

The Solver menu command is ideally suited for determining the maximum profit, and with some forethought, can also identify the breakeven points. As described in Template 2.4 menu command, it does not provide a graph of the results. The only difference between Template 2.4 and Template 2.5 is the addition of cell B11.

Figure 2-5 displays the Solver dialogue box. The constraint cell, B11, limits the analysis to the region where profit is positive. The endpoints of this region are the breakeven points, so these values are returned automatically as part of the output. Figure 2-6 shows the dialog box that appears when Solver finds a solution. Note that cell A14 now contains the number of units to maximize profit and the corresponding maximum profit value appears in cell D14.

The Answer Table for this problem appears in Figure 2-7. This is just one of the reports that can be generated using the Solver Answer Dialog Box.

Present Economy Non Linear Breakeven Analysis						Template 2.5
Total Revenue		0				
Function Constraint		1				
a =	$180.00			KEY:		
b =	$0.02				=	User input
Total Cost Function					=	Unique Formula
Constraint						
Fixed =	$73,000.00			Unique Formulas		
Variable =	$83.00			B14		B$5*A14-B$6*A14^2
constraint cell =	0			C14		B$9+B$10*A14
Units Demanded	Total	Totaal Costs	Profit	D14		B14-C14
(D)	Revenue			B11		IF(D14<0:C3;C4)
0	0	73000	-73000			

Figure 2-5. Template 2.5 Solver Definition Dialog box.

Present Economy Non Linear Breakeven Analysis							Template 2.5	
Total Revenue		0						
Function Constraint		1						
a =	$180.00			KEY:				
b =	$0.02				=	User input		
Total Cost Function					=	Unique Formula		
Constraint								
Fixed =	$73,000.00			Unique Formulas				
Variable =	$83.00			B14		B$5*A14-B$6*A14^2		
constraint cell =	1			C14		B$9+B$10*A14		
Units Demanded	**Total**	**Totaal Costs**	**Profit**	D14		B14-C14		
(D)	**Revenue**			B11		IF(D14<0:C3;C4)		
2424.999953	318887.4961	274274.9961	44612.5					

Figure 2-6. Solver Answer box for Template 2.5.

Microsoft Excel 8.0a Answer Report
Worksheet: [FIG2.xls]Sheet1
Report Created: 04-02-1998 12:44:50

Target Cell (Max)

Cell	Name	Original Value	Final Value
$D$14	Profit	-73000	44612.5

Adjustable Cells

Cell	Name	Original Value	Final Value
$A$14	(D)	0	2424.999953

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$B$11	constraint cell =	1	$B$11>=0	Not Binding	1

Chapter 3
Principles of Money-Time Relationships

Tenth Edition Highlights

Typical engineering projects span multiple years and involve sizable cash flows. Because money has a time value, the cash flows associated with each alternative cannot be directly compared unless they occur at the same point in time. This chapter presents the notation, definitions, and relationships that allow one to convert cash flows into equivalent amounts at different points in time.

Chapter 3 provides the foundation upon which the remainder of the text builds. Students experiencing difficulty in the course are invariably those who do not master these basics. The best advice for doing well in the course is to use a cash flow diagram to set up every problem, no matter how simple it may appear. One must also understand how the cash flows in annuities and gradients relate to the quantities *P* and *F*.

The most difficult aspect when working with "real" problems, such as mortgage payments, is ensuring that the interest rate and time period are stated in the same terms. Sections 3.16, 3.17, and 3.18 deserve close attention by those who will apply this material to practical problems.

The following notation and formula summaries capture the major points of the chapter.

Notation Summary

- P: Single cash flow at time 0 (beginning of period *N*).
- F: Single cash flow at the end of study (end of period *N*).
- A: Equal, end of period cash flow series starting at the end of period 1 and terminating at the end of period *N*.
- G: Arithmetic: A cash flow series starting at the end of period 2, increasing or decreasing by a constant, fixed amount and terminating at the end of period *N*.
- G: Geometric: A cash flow series starting at the end of period 1, increasing or decreasing by a fixed percentage and terminating at the end of period *N*.
- N: Number of time periods in study.
- M: Number of compounding periods per year.
- i: Constant periodic interest rate.
- r: Nominal annual interest rate.
- $\underline{I}$: Total simple interest.

Formula Summary

Cash flows and interest may be discrete or continuous. The formulas in the text are summarized by these categories in Table 3.1. Formulas pertinent to interest calculations follow:

Total simple interest earned on *P* for *N* periods at *i*% per period

$$\underline{I} = PNi$$

Total amount repaid at the end of *N* periods based on simple interest

$$F = P + \underline{I}$$

Convenience rate for geometric gradients, discrete compounding

$$i_{CR} = \frac{1+i}{1+\bar{f}} - 1$$

Effective annual interest rate for compounding more often than once per year

$$i = \left(1 + \frac{r}{M}\right)^M - 1$$

Table 3.1. Summary of formulas.

Cash Flow:	**Discrete**	**Discrete**	**Continuous**
Interest:	***Discrete***	***Continuous***	***Continuous***
F/P	$F = P(1+i)^N$	$F = Pe^{rN}$	
P/F	$P = F(1+i)^{-N}$	$P = Fe^{-rN}$	
F/A	$F = A\left[\frac{(1+i)^N - 1}{i}\right]$	$F = A\frac{e^{rN} - 1}{e^r - 1}$	$F = A\frac{e^{rN} - 1}{r}$
P/A	$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]$	$P = A\frac{e^{rN} - 1}{e^{rN}(e^r - 1)}$	$P = A\frac{e^{rN} - 1}{re^{rN}}$
A/F	$A = F\left[\frac{i}{(1+i)^N - 1}\right]$	$A = F\frac{e^r - 1}{e^{rN} - 1}$	$\overline{A} = F\frac{r}{e^{rN} - 1}$
A/P	$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right]$	$A = P\frac{e^{rN}(e^r - 1)}{e^{rN} - 1}$	$\overline{A} = P\frac{re^{rN}}{e^{rN} - 1}$
F/G (arithmetic)	$F = \frac{G}{i}\left[\frac{(1+i)^N - 1}{i}\right] - \frac{NG}{i}$		
A/G (arithmetic)	$A = G\left[\frac{1}{i} - \frac{N}{(1+i)^N - 1}\right]$		
P/G (arithmetic)	$P = G\left\{\frac{1}{i}\left[\frac{(1+i)^N - 1}{i(1+i)^N} - \frac{N}{(1+i)^N}\right]\right\}$		
P/G (geometric)	$P = \frac{A_1}{1+\overline{f}}\left[\frac{(1+i_{CR})^N - 1}{i_{CR}(1+i_{CR})^N}\right]$		

Overview

The Chapter 3 spreadsheet templates reinforce the concepts in the Tenth Edition of *Engineering Economy* (DeGarmo, Sullivan, Bontadelli, and Wicks) and provide a vehicle for exploring the fundamental relationships in greater depth. Differences between problem formulation for the closed form approach and the spreadsheet approach are also reviewed. The greatest differences are evident when dealing with annuities and gradients. The templates also illustrate fundamental spreadsheet modeling techniques which will be utilized in later chapters to build more complex models.

Spreadsheet Templates

3.1	simpint	Simple vs. compound interest
3.2	inttable	Table of factors for discrete cash flows, discrete compounding
3.3	inttblc	Table of factors for discrete cash flows, continuous compounding
3.4	inttbld	Table of factors for continuous cash flows, continuous compounding.
3.5	intrate	Illustration of (P/A,i,N) as a function of i and N
3.6	annuity	PW of an annuity
3.7	gradient	Arithmetic vs. geometric gradients
3.8	varyint	Nonconstant periodic interest rates
3.9	effint	Effective interest rate for compounding frequency >1 per year
3.10	multcmpd	Multiple compounding periods between cashflows

New Features and Functions

NPV(i,range)

Computes present value of any arbitrary cash flows in the specified range. Caution is required for handling the cash flow (P) at $T = 0$. Let

P_range = range containing the cash flows at the end of periods 1, 2, ..., N

F_range = range containing the cash flows at the end of periods 0, 1, ..., N

Then the PW can be determined using either formula:

PW = P + NPV(i, *P_range*) or

PW = NPV(i, *R_range*) * (1 + i)

PMT(P,i,N)

Returns the annuity equivalent to P taken over N periods.

SUM(range)

Returns the sum of the values in the cells of the range specified. Verify how your software treats text labels included in the range to avoid surprises.

QUOTIENT(x,y)

Returns [x/y], i.e., the integer portion (without rounding) of x/y.

EXP(x)

Returns e^x.

Financial Function Assumptions

- End of period convention.
- i is constant and expressed in decimal form
- The time between adjacent cash flows is constant.
- The time units of i and N must agree.
- The first cash flow in a range occurs at the end of period 1.
- A is constant.
- F occurs at the end of period N.

Template 3.1: Simple vs. Compound Interest

The amount of simple interest received on a deposit P is a function of i and N. In a coordinate system based on:

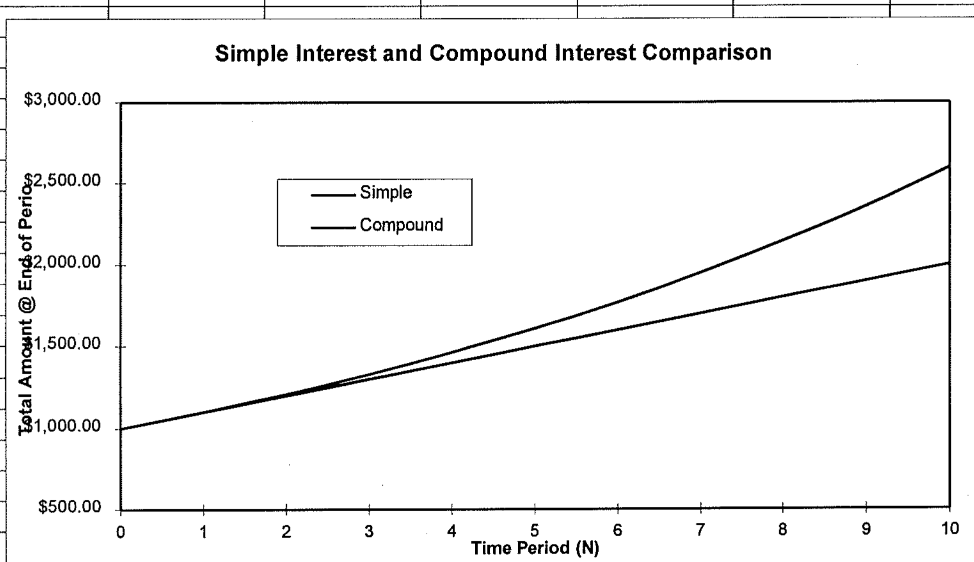
$$(N, \text{amount due at } N)$$

the relationship is linear with slope Pi, starting at the point $(0,P)$. The amount due at the end of any period is $(N,P(1+iN))$.

Compound interest is a function of the amount on deposit, in addition to the interest generated up to that point in time. This function is nonlinear and will always be larger than the simple interest amount for any positive nonzero value N. The difference between simple and compound interest becomes larger as both i and N increase.

In Template 3.1, the two interest formulas are entered into cells B8 and C8. These cells are copied down their respective columns. Note that absolute addressing is used to reference the values of i and P, while relative addressing is used to refer to N. The range B8..C18 is plotted.

	A	B	C	D	E	F	G
1	Simple Interest and Compound Interest Comparison					Template 3.1	
2							
3	i =	10.00%					
4	P =	$1,000					
5							
6		Total At End of Period (N)					
7	period (N)	Simple	Compound				
8	0	$1,000.00	$1,000.00				
9	1	$1,100.00	$1,100.00				
10	2	$1,200.00	$1,210.00				
11	3	$1,300.00	$1,331.00				
12	4	$1,400.00	$1,464.10				
13	5	$1,500.00	$1,610.51				
14	6	$1,600.00	$1,771.56				
15	7	$1,700.00	$1,948.72				
16	8	$1,800.00	$2,143.59				
17	9	$1,900.00	$2,357.95				
18	10	$2,000.00	$2,593.74				
19							
20	KEY:						
21		= User Input					
22		= Unique Formula					
23							
24	Unique Formulas						
25	B8	=$B$4*A8$B$3+$B$4					
26	C8	=$B$4*(1+$B$3)^A8					
27							



Template 3.2: Simple vs. Compound Interest

From time to time, it is helpful to have tables for values not included in Appendix C in the Tenth Edition of *Engineering Economy*. Template 3.2 creates a discrete compounding table for any value entered into cell B5. The start and increment cells (B3 and B4) permit one to focus on a narrow range of values, rather than complete a full table.

While it is helpful to have a compounding table for a specific interest rate, the real value of this template is in illustrating the relationship between the various compounding factors. The factor $(1+i)^N$ forms the basis for all compound interest formulas. For any value of N, it is only necessary to calculate this quantity once and then refer to it in all other formulas. The complexity of the equations can be further reduced by recognizing that $(P/F,i,N)$, $(A/F,i,N)$ and $(A/P,i,N)$ are the respective inverses of $(F/P,i,N)$, $(A/F,i,N)$ and $(P/A,i,N)$. Making use of these relationships results in an increase of computational efficiency of over 24% versus using the formulas as provided in Table 3.1.

Some may find it beneficial to plot the six factors over a short range to more easily visualize the relationship between the factors.

	A	B	C	D	E	F	G	H
1	Discrete Cash Flow, Discrete Compounding Factors					Template 3.2		
2								
3	Start:	1						
4	Increment:	1						
5	i =	5%						
6								
7	N	(F/P,i,N)	(P/F,i,N)	(F/A,i,N)	(A/F,i,N)	(P/A,i,N)	(A/P,i,N)	
8	1	1.0500	0.9524	1.0000	1.0000	0.9524	1.0500	
9	2	1.1025	0.9070	2.0500	0.4878	1.8594	0.5378	
10	3	1.1576	0.8638	3.1525	0.3172	2.7232	0.3672	
11	4	1.2155	0.8227	4.3101	0.2320	3.5460	0.2820	
12	5	1.2763	0.7835	5.5256	0.1810	4.3295	0.2310	
13	6	1.3401	0.7462	6.8019	0.1470	5.0757	0.1970	
14	7	1.4071	0.7107	8.1420	0.1228	5.7864	0.1728	
15	8	1.4775	0.6768	9.5491	0.1047	6.4632	0.1547	
16	9	1.5513	0.6446	11.0266	0.0907	7.1078	0.1407	
17	10	1.6289	0.6139	12.5779	0.0795	7.7217	0.1295	
18								
19								
20	KEY:							
21		= User Input						
22		= Unique Formula						
23								
24	Unique Formulas							
25	A8	=B3						
26	A9	=A8+$B$4						
27	B8	=(1+$B$5)^A8						
28	C8	=1/B8						
29	D8	=(B8-1)/$B$5						
30	E8	=1/D8						
31	F8	=(B8-1)/($B$5*B8)						
32	G8	=1/F8						
33								
34								

Template 3.3: Table of Factors for Discrete Cash Flows, Continuous Compounding

This template uses the same approach described in Template 3.2 to create tables for discrete cash flows sand continuous compounding. The only difference between the tables is the substitution of the term e^{rN} for $(1+i)^N$. This template provides more comprehensive tables than those in Appendix D of the Tenth Edition of *Engineering Economy*.

	A	B	C	D	E	F	G	H
1	Discrete Cash Flow, Continuous Compounding Factors					Template 3.3		
2								
3	Start:	1						
4	Increment:	1						
5	r =	5%						
6								
7	N	(F/P,_r_,N)	(P/F,_r_,N)	(F/A,_r_,N)	(A/F,_r_,N)	(P/A,_r_,N)	(A/P,_r_,N)	
8	1	1.0513	0.9512	1.0000	1.0000	0.9512	1.0513	
9	2	1.1052	0.9048	2.0513	0.4875	1.8561	0.5388	
10	3	1.1618	0.8607	3.1564	0.3168	2.7168	0.3681	
11	4	1.2214	0.8187	4.3183	0.2316	3.5355	0.2828	
12	5	1.2840	0.7788	5.5397	0.1805	4.3143	0.2318	
13	6	1.3499	0.7408	6.8237	0.1465	5.0551	0.1978	
14	7	1.4191	0.7047	8.1736	0.1223	5.7598	0.1736	
15	8	1.4918	0.6703	9.5926	0.1042	6.4301	0.1555	
16	9	1.5683	0.6376	11.0845	0.0902	7.0678	0.1415	
17	10	1.6487	0.6065	12.6528	0.0790	7.6743	0.1303	
18								
19								
20	KEY:							
21		= User Input		NOTE:	_r_ corresponds to the underscore 'r' in the text			
22		= Unique Formula						
23								
24	Unique Formulas							
25	A8	=B3						
26	A9	=A8+$B$4						
27	B8	=EXP($B$5*$A8)						
28	C8	=1/B8						
29	D8	=(B8-1)/(EXP($B$5)-1)						
30	E8	=1/D8						
31	F8	=(B8-1)/(B8*(EXP($B$5)-1))						
32	G8	=1/F8						
33								
34								
35								

Template 3.4: Table of Factors for Continuous Cash Flows, Continuous Compounding

Template 3.4 provides a more comprehensive table than Appendix D of the Tenth Edition of *Engineering Economy.*

The template also introduces the use of the "hidden" cell format. This is useful to prevent formulas or comments from cluttering the screen or printed output. The contents are still visible when the cursor is placed in the cell containing the hidden material.

Cell F8 is highlighted, indicating that a formula is present, but nothing is visible in that column. Since the quantity e^{rN} appears in each quantity, it is calculated separately and then referenced in the other formulas. Since this quantity is not part of the table, the hidden format is used for the range F8..F17.

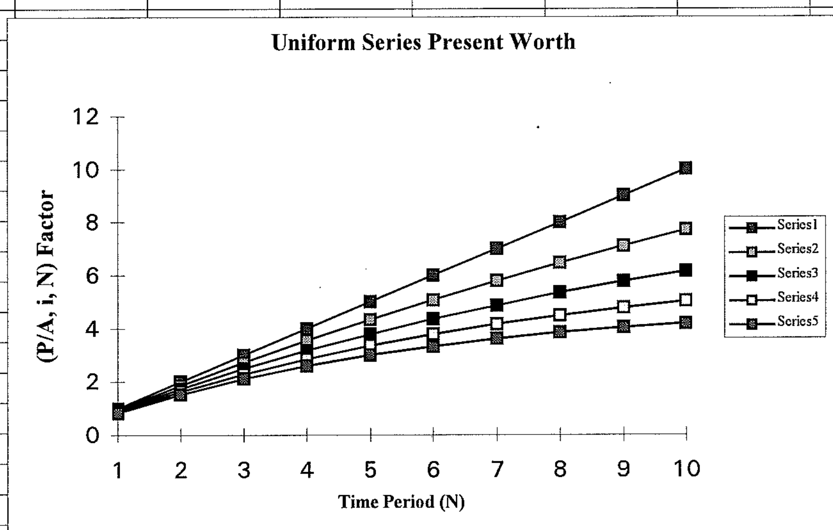
	A	B	C	D	E	F	G	H
1	Continuous Cash Flow, Continuous Compounding Factors					Template 3.4		
2								
3	Start:	1						
4	Increment:	1						
5	r =	5%						
6								
7	N	(F/A~,_r_,N)	(A~/F,_r_,N)	(P/A~,_r_,N)	(A~PF,_r_,N)			
8	1	1.0254	0.9752	0.9754	1.0252			
9	2	2.1034	0.4754	1.9033	0.5254			
10	3	3.2367	0.3090	2.7858	0.3590			
11	4	4.4281	0.2258	3.6254	0.2758			
12	5	5.6805	0.1760	4.4240	0.2260			
13	6	6.9972	0.1429	5.1836	0.1929			
14	7	8.3814	0.1193	5.9062	0.1693			
15	8	9.8365	0.1017	6.5936	0.1517			
16	9	11.3662	0.0880	7.2474	0.1380			
17	10	12.9744	0.0771	7.8694	0.1271			
18								
19								
20	KEY:							
21		= User Input		NOTE:	_r_ corresponds to the underscore 'r' in the text			
22		= Unique Formula			A~ corresponds to the overscore A in the text			
23								
24	Unique Formulas							
25	A8	=B3						
26	A9	=A8+$B$4						
27	B8	=(F8-1)/$B$5						
28	C8	=1/B8						
29	D8	=(F8-1)/($B$5*F8)						
30	E8	=1/D8						
31	F8	=EXP($B$5*A8)						
32								
33								
34								
35								

Template 3.5 : Illustration of (P/A,*i*, *N*) as a Function of *i* and *N*

This template permits one to virtually examine the effect of the values of *i* and *N* on the various compounding factors. Familiarity with the factors values is beneficial for performing "reality checks" on your solutions.

The *uniform series present worth factor* is used in this case, but the template will work with any other factor placed in cell B6. Mixed addressing is used so that one formula can be used to complete the table, even though two variables are present. The range B6..F15 is plotted.

	A	B	C	D	E	F	G	H
1	**UNIFORM SERIES PRESENT WORTH (P/A, i, N)**						**TEMPLATE 3.5**	
2								
3								
4			**Discount Rate (i)**					
5	**Period (N)**	0%	5.00%	10.00%	15%	20.00%		
6	1	0.99990001	0.952380952	0.909090909	0.869565217	0.833333333		
7	2	1.99970004	1.859410431	1.73553719	1.625708885	1.527777778		
8	3	2.9994001	2.723248029	2.486851991	2.283225117	2.106481481		
9	4	3.9990002	3.545950504	3.169865446	2.854978363	2.588734568		
10	5	4.99850035	4.329476671	3.790786769	3.352155098	2.99061214		
11	6	5.99790056	5.075692067	4.355260699	3.784482694	3.325510117		
12	7	6.99720084	5.786373397	4.868418818	4.160419734	3.604591764		
13	8	7.9964012	6.463212759	5.334926198	4.487321508	3.837159803		
14	9	8.99550165	7.107821676	5.759023816	4.77158392	4.030966503		
15	10	9.994502199	7.721734929	6.144567106	5.018768626	4.192472086		
16								
17	**KEY:**							
18		= User Input						
19		=Unique Formula						
20								
21	**Unique Formulas**							
22	B6:	((1+B$5)^$A6-1)/(B$5*(1+B$5)^$A6)						
23								
24								
25								
26								
27								
28								
29								
30								
31								
32								
33								
34								
35								
36								
37								
38								
39								
40								
41								
42								



Template 3.6: Present Worth (PW) of an Annuity

This template illustrates three different methods for handling annuities in spreadsheets. The results are the same regardless of the method selected, so the selection between approaches should be based on convenience.

The first approach brings each end of period cash flow to the present by multiplying it by (P/F, *i*, *N*). The result values appear in ranges C8..C12. Since the values in this range all occur at time $k = 0$, they can be added together to determine the Present Worth (PW) using the sum function in cell C14. Note that this method requires the cash flow for each period to appear in a separate cell.

The second approach uses the closed form formula in the text. Cell D14 contains a formula to multiply A* (P/A, *i*, *N*).

The third approach, and the one that is generally most convenient, is to use the NPV function. When dealing with annuities, one copies the value of A to each period, and then applies the function to the range. In this approach, the range that contains the annuity is B8..B12.

The NPV function is the most flexible of all financial functions. Modeling is simplified by computing the PW of any cash flow, and then converting that measure to FW or AW, if needed.

When solving problems with closed form analysis, deferred annuities are cause or concern. Once the amounts are listed in the proper period, no special action is required to handle deferred annuities in spreadsheets, other than placing the annuity in the proper cell.

	A	B	C	D	E	F
1	**ANNUITIES**				**TEMPLATE 3.6**	
2						
3	**Annuity**	$250				
4	**Amount =** i =	6.00%				
5						
6	**Period (N)**	Cash Flow	P/F	P/A		
7	0					
8	1	$250	$235.85			
9	2	$250	$222.50			
10	3	$250	$209.90			
11	4	$250	$198.02			
12	5	$250	$186.81			
13						
14	**PW of Annuity =**	$1,053.09	$1,053.09	$1,053.09		
15						
16						
17	**KEY:**					
18						
19						
20						
21	**Unique Formulas**					
22	B8	=B				
23	B14	3=NPV(B4,B8:B12				
24	C8	)=B8/(1+$B$4)^				
25	C14	A8SUM(C8..C12)				
26	D14	=((1+B4)^A12-				
27		1)/(B4*(1+B4)^A12)*B3				

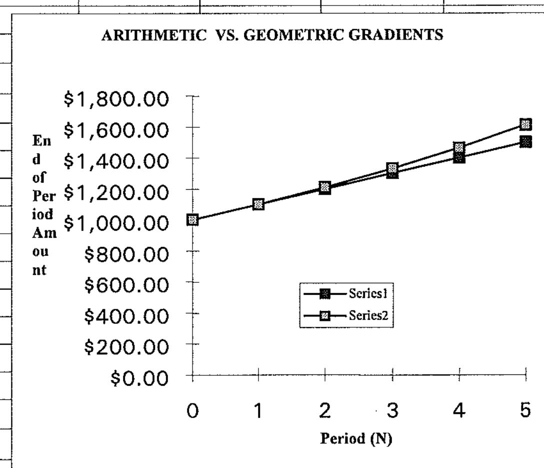
Template 3.7 : Arithmetic vs. Geometric Gradients

Gradients are handled in the same fashion as annuities in spreedsheets. The base value is modified either arithmetically or geometrically to fill the cash flow range and then standard functions are applied to determine the Present Worth (PW). The spreadsheet approach simplifies analysis of gradients.

The formula for generating an arithmetic gradient appears in cell B11; the formula for a geometric gradient appears in cell C11.

Note the difference in PW for each type of gradient in row 17. The reason for the difference is evident by looking at the graph. The difference in PW increases for larger values of *N* and *i.*

	A	B	C	D	E	F	G	H
1	**GRADIENTS**					**TEMPLATE 3.7**		
2								
3	**Base Amount =**	$1,000.00						
4								
5								
6		Type of Gradient:						
7		Arithmetic	Geometric					
8		$100.00	10%					
9	**Period (N)**							
10	0	$1,000.00	$1,000.00					
11	1	$1,100.00	$1,100.00					
12	2	$1,200.00	$1,210.00					
13	3	$1,300.00	$1,331.00					
14	4	$1,400.00	$1,464.10					
15	5	$1,500.00	$1,610.51					
16								
17	**PW =**	$5,856.05	$6,000.00					
18								
19								
20	**KEY:**							
21								
22								
23								
24	**Unique Formulas**							
25	B10	=$B$3						
26	B11	=B10+$B$8						
27	C11	=C10+C10*$C$8						
28	B17	=NPV(0.1,B11..B15)+$B$3						
29								



Template 3.8: Nonconstant Periodic Interest Rates

When the assumptions for the functions are not met, one must break the problem down into a series of subproblems where the assumptions do hold. This template determines the future amount of any series of cash flows with varying periodic interest rates. The data from example 3.19 in the text is used for illustration.

Cell D5 takes the ending balance from period N -1 and adds any additional borrowing that occurs at the beginning of period N. Cell E5 multiplies the total amount owed at the beginning of the period by the interest rate for that period. Next, cell F5 combines the amount owed at the beginning of the period with the interest for that period. The last cell in the table, F8, contains the final answer.

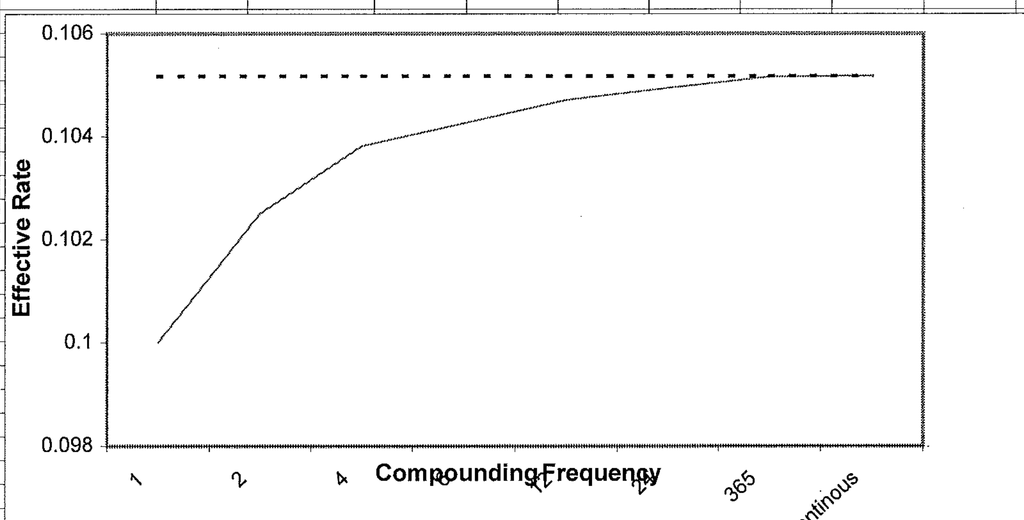
	A	B	C	D	E	F	G
1	Nonconstant	Interest rates		**Template 3.8**			
2							
3	Period	Interest Rate(I)	Additional Borrowing	Beginning of Period	Interest Due for Period	End of Period Balance	
4	0					$1,000.00	
5	1	10.00%	$0.00	$1,000.00	$100.00	$1,100.00	
6	2	12.00%	$0.00	$1,100.00	$132.00	$1,232.00	
7	3	12.00%	$1,000.00	$2,232.00	$267.84	$2,499.84	
8	4	14.00%	$0.00	$2,499.84	$349.98	$2,849.82	
9							
10							
11							
12	KEY:						
13		=User Input					
14		=Unique Formula					
15							
16							
17							
18	Unique Formulas						
19	D5	F4+C5					
20	E5	D5*B5					
21	F5	E5+D5					
22							
23							
24							
25							

Template 3.9: Effective Interest Rate for Compounding Frequency. 1 per Year

Interest is generally compounded more often than once per year. As a result, the actual rate paid annually is greater than the stated annual rate. This template provides a table of the effective interest rate for several compounding frequencies.

The graph illustrates the diminishing effect of more frequent compounding. The horizontal line represents the continuos compounding rate. To plot reference line in the spreadsheet, copy the constant value over a range and plot the range (C4: C11). If desired, this range can be formatted using hidden format.

	A	B	C	D	E	F	G	H	I	J	K
1	Effective Interest Rate					**Template 3.9**					
2											
3	Compounding frequency per Year	10.00%									
4	1	10.00%	10.52%								
5	2	10.25%	10.52%								
6	4	10.38%	10.52%								
7	6	10.43%	10.52%								
8	12	10.47%	10.52%								
9	24	10.49%	10.52%								
10	365	10.52%	10.52%								
11	continous	10.52%	10.52%								
12											
13	KEY:										
14		=User Input									
15		=Unique Formulas									
16											
17	Unique Formulas										
18	B4	(1+B$3/$A4)^$A4-1									
19	C12	EXP(C$2)-1									
20	D5	$C$11									
21											



Template 3.10: Multiple Compounding Periods between Cash Flow

In many applied problems the interval between cash flows and the compounding period is different. To satisfy the basic engineering economy assumptions, one must reexpress either the time interval between cashflows to agree with the interval between interest compounding periods or reexpress the interest-compounding period to agree with the cash flows. This template illustrates both approaches. Consider the problem of deciding what amount A can be withdrawn annually for four years from a savings account that pays 12% compound annually. The first withdrawal occurs one year after the initial deposit and the final withdrawal, which results in a balance of zero, occurs at the end of four years.

In Approach 1, the problem is solved on a monthly basis and then the monthly values are expressed on an annual basis. Cell B6 determines the monthly rate by deciding the stated annual rate by the number of compounding periods per year. Cell B12 uses this interest rate and the PMT function to calculate an equivalent annuity, based on 46 (4 years* 12 months/year) periods. So at 1% per period, $1000 at time k=0 is equivalent to 48 end of period cash flows of $26.35 each.

The equivalent annual amount id determined in cell C27 by computing the PW for one year and then bringing that forward to the end of the year. All compounding id based on the monthly rate. Thus, $333.98 annually is equivalent to $26.33 monthly. The formulas in cells B65 and C65 confirm this.

The function in cell D16 makes it easier to identify in which year a particular cash flow occurs. This is done in cell B7, which computes the effective interest rate. Cell G16 computes the annual cash flow based on this rate. Cell G21 confirms that the answer is the same as with Approach 1. Approach 2 involes fewer steps and is generally the preferred method.

	A	B	C	D	E	F	G	H	I	J
1	Multiple Compounding Periods Between Cash Flows		**Template 3.10**							
2										
3	Nominal:	12.00%								
4	Periods/Yr:	12								
5	P =	$1,000.00								
6	i:	1.00%								
7	i(eff):	12.68%								
8										
9	Approach 1					Approach 2				
10	Use compounding					Use cash flows,				
11	period & rate					change period				
12	change cash flows					and rate				
13										
14	monthly A =	($26.33)								
15										
16	Period (Month)	Month(A)	Annual(A)	End of Year		Period(year)				
17	0					0				
18	1	($26.33)		0		1	($333.98)			
19	2	$26.33		0		2	($333.98)			
20	3	$26.33		0		3	($333.98)			
21	4	$26.33		0		4	($333.98)			
22	5	$26.33		0						
23	6	$26.33		0		PW =	($1,000.00)			
24	7	$26.33		0						
25	8	$26.33		0						
26	9	$26.33		0		KEY:				
27	10	$26.33		0			=User Input			
28	11	$26.33		0			=Unique Formula			
29	12	$26.33	275.1749301	1						
30	13	$26.33		1		Unique Formulas				
31	14	$26.33		1		B6	B3/B4			
32	15	$26.33		1		B7	(1+B6)6B4-1			
33	16	$26.33		1		B14	PMT(B6,48,B5)			
34	17	$26.33		1		B18	B14			
35	18	$26.33		1		D18	MOD(A17,12)			
36	19	$26.33		1		C29	NPV($B$6,B18:B29)*(1+$B$6)^12			
37	20	$26.33		1		B67	NPV(B6,B18:B65)			
38	21	$26.33		1		C67	NPV(B6,C18:C65)			
39	22	$26.33		1		G18	PMT(B$7,F$21,B$5)			
40	23	$26.33		1		G23	NPV(B24,G35:G38)+G34			
41	24	$26.33	333.9303043	2						
42	25	$26.33		2						
43	26	$26.33		2						
44	27	$26.33		2						
45	28	$26.33		2						
46	29	$26.33		2						
47	30	$26.33		2						
48	31	$26.33		2						
49	32	$26.33		2						
50	33	$26.33		2						
51	34	$26.33		2						
52	35	$26.33		2						
53	36	$26.33	333.9303043	3						
54	37	$26.33		3						
55	38	$26.33		3						
56	39	$26.33		3						
57	40	$26.33		3						
58	41	$26.33		3						
59	42	$26.33		3						
60	43	$26.33		3						
61	44	$26.33		3						
62	45	$26.33		3						
63	46	$26.33		3						
64	47	$26.33		3						
65	48	$26.33	333.9303043	4						
66										
67	pw=	$947.711942	$1,244.810906							
68										
69										

Chapter 4
Basic Methods for Evaluating a Project

Tenth Edition Highlights

Engineering economy problems reduce to a simple question: "How much are you willing to invest now in order to generate savings (or reduce expenses) in the future?" Alternatives differ in the magnitude and timing of their cash flows. Because money has a time value, cash flows occurring at different points in time are not directly comparable.

To compare alternatives properly, one must reexpress the cash flows either at a common point in time, or over a common time interval. Once a common measure of merit is selected, these values are compared and a selection made. Measures of merit fall into three general categories: (a) equivalent worth, (b) rate of return, and (c) "other."

This chapter presents the basic measures of merit. Chapter 5 in the Tenth Edition of *Engineering Economy* (DeGarmo, Sullivan, Bontadelli, and Wicks) describes the decision rules to select between alternatives.

EQUIVALENT WORTH

Any arbitrary (but common) point in time can be selected to make the comparison. However, the most common times are the present (year $k = 0$; the start of the project) and the future (the end of period N; the conclusion of the project). The corresponding measures of merit are the Present Worth (PW) and the Future Worth (FW).

Cash flows can also be expressed as a series of equal amounts over a period of time. Although any period can be used, the study period (N) is customary. This measure of merit is the Annual Worth (AW).

Any equivalent worth measure can be converted to any other by the application of the appropriate discount factor. Calculation of equivalent worth measures requires that the MARR be known.

RATE OF RETURN

Rate of Return methods provide a different view of an alternative' s financial performance. Knowledge of the MARR is not required to compute the rate of return, but is required to make a selection. This point is addressed in Chapter 5.

The Internal Rate of Return (IRR) is the periodic interest rate required for the PW (or other equivalent worth measure) of the cash flow to be zero. The IRR allows comparison of more diverse alternatives than the equivalent worth methods. However, except for special cases, manual computations are difficult, multiple answers may result, and the IRR may lead to different ranking of alternatives than the equivalent worth methods. These issues can be resolved easily, as shown in Chapter 5.

The External Rate of Return (ERR) uses an external interest rate to move all negative cash flows to year $k = 0$ and all positive cash flows to year $k = N$. This simplifies the calculation of the rate of return, and results in a unique answer.

"OTHER"

The "other" category includes the Payback method, which is very common in practice. The payback period is the time required to recover the initial investment. This method does not consider the time value of money and may result in short term profitability at the expense of long term survival. It is a good measure of risk, but should be supplemented with one of the previous measures of merit. The discounted payback method considers the time value of money, but still suffers many of the same problems as the payback method.

The preceding measures of merit constitute the basic tools for all engineering economic analysis. This material builds on the fundamental relationships established in Chapter 3, and the combined material is refined in Chapter 5 to establish a formal decision making process.

The measure of merit responsible for the greatest number of errors in the classroom is Annual Worth. Mastering the definitions and relationships of A, P, G, and F are usually sufficient to overcome this problem. Since AW is an equivalent worth method, one can calculate the PW and then multiply by the factor (A/P,i,N) to check the AW solution.

Notation Summary

k	Period index
F_k	Single future amount occurring at the end of period k. ($F_0 = P$). If F appears without a subscript, it is implied to be F_N.
R_k	Single amount (revenue) occurring at the end of period k.
E_k	Single amount (expense) occurring at the end of period k.
$\underline{R}$	Equivalent periodic revenues
$\underline{E}$	Equivalent periodic expenses
ϵ	The external per period interest rate, used with the external rate of return method.
S	Salvage (market) value at N.
I	Initial investment = PW of all capital investments.

Bond problems

Z	Face or par value
C	Redemption value (usually = Z)
r	Nominal bond rate per period
N	Number of periods before redemption
i	Bond yield rate per period
V_N	Value of bond N periods prior to redemption

Formula Summary

The measures of merit introduced in this chapter are summarized by category in Table 4.1. Any of the CR formulas can be used to compute AW.

Bond problems have their own special terminology. The formula for bond value is given below.

$$V_N = C(P/F,i,N) + rZ(P/A,i,N)$$

Overview

Speed of calculation is an obvious advantage of electronic spreadsheets compared to the closed form approach given in the text. What may not be so obvious is that spreadsheets also have the advantage in problem formulation. Since problem formulation ultimately dictates the reliability of the solution, this advantage should be seized whenever possible.

The closed form solution approach requires that one break the cash flows into a series of subproblems that fit the given assumptions and formulas. Arithmetic gradients provide an excellent example of the difficulties encountered with some closed form modeling. The arithmetic gradient formulas are for increasing gradients only and further assume that the first gradient amount occurs at the end of period 2. If these assumptions are not met, as in the case of a decreasing arithmetic gradient, the analyst must reexpress the decreasing gradient as the difference between an annuity and an increasing arithmetic gradient. Errors are easy to make in such situations.

While one must still be aware of basic assumptions, spreadsheet model formulation is decidedly less cumbersome. The cash flows are entered into a range, such that the period of time between cash flows (cells) is constant. In the case of a deferred annuity, zeros are entered as place holders in the time periods without cash flows. Note that with this approach, there is no need to compute the Capital Recovery amount directly.

Table 4.1. Formula summary.

	Single Point	Period
Equivalent Worth	$PW(i\%) = \sum_{k=0}^{N} F_k (1+i)^{-k}$ $FW(i\%) = \sum_{k=0}^{N} F_k (1+i)^{N-k}$	$AW(i\%) = \underline{R} - \underline{E} - CR(i\%)$ $CR(i\%) = I(A/P,i,N) - S(A/F,i,N)$ $CR(i\%) = (I-S)(A/F,i,N) + I(i)$ $CR(i\%) = (I-S)(A/P,i,N) + S(i)$
Rate of Return		*IRR*: $PW = \sum_{k=0}^{N} R_k(P/F,i',k) - \sum_{k=0}^{N} E_k(P/F,i',k) = 0$ *ERR*: $\sum_{k=0}^{N} E_k(P/F,\epsilon,k)(F/P,i',N) = \sum_{k=0}^{N} R_k(F/P,\epsilon,N-k)$
Other		*Payback*: $\sum_{k=1}^{\theta} (R_k - E_k) - I \geq 0$ *Discounted Payback*: $\sum_{k=1}^{\theta'} (R_k - E_k)(P/F,i,k) - I \geq 0$

If revenues and expenses come from several sources, each with different timing, the approach illustrated in Figure 4.1 works well. Create a column for each type of cash flow and enter the appropriate values for each period. Do a row sum (SUM) for each period to get the net cash flow. Use the NPV function to compute the PW of the net cash flow. The PW value can be easily converted to FW or AW by multiplying by (F/P,i,N) or (A/P,i,N) respectively. The IRR function can also be applied to the resultant cash flow range, provided there is a single change of sign.

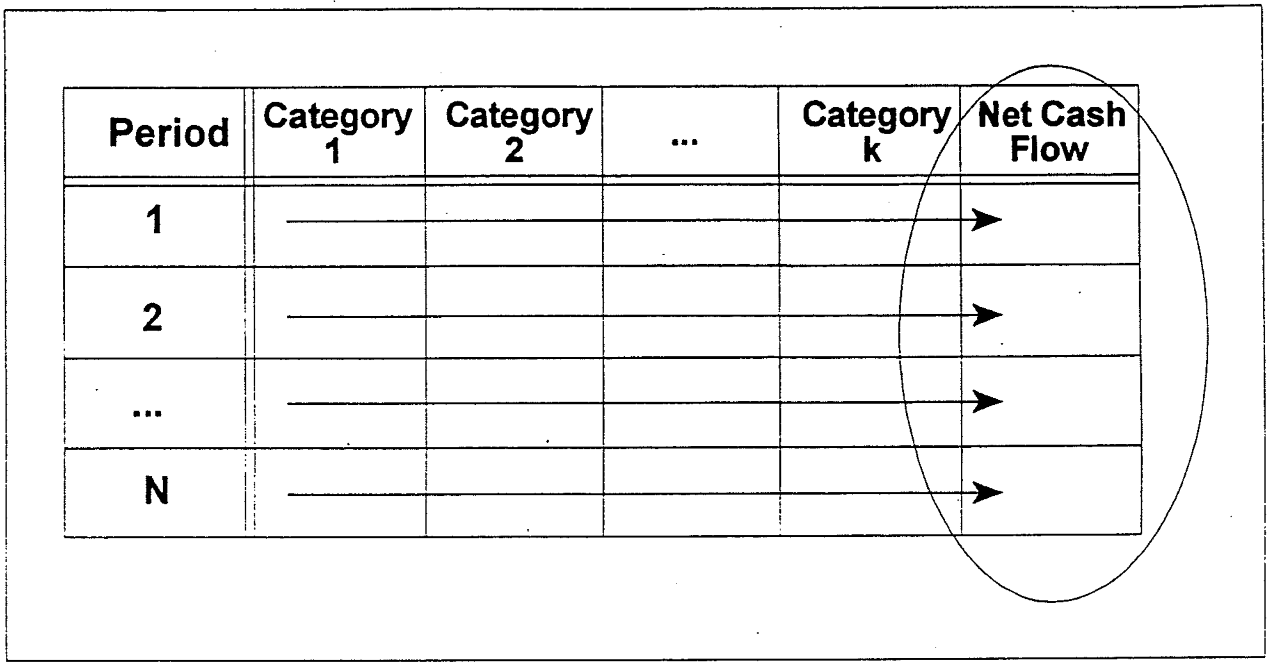


Figure 4.1. Spreadsheet model for cash flows. The Net Cash Flow column contains the net periodic cash flow used to compute the measure of merit.

Spreadsheet Templates

4.1	msmerit	Measures of merit (PW, AW, FW, IRR)
4.2	caprecvr	Capital Recovery amount
4.3	irrgraf	Graphic determination of IRR
4.4	err	External rate of return
4.5	payback	Regular (simple) and discounted payback

New Features and @Functions

IRR(guess,range)

Returns the Internal Rate of Return (IRR) for cash flows in the specified range. There should be only one change of sign in the cash flow. The *guess* argument is a starting point used by the internal algorithm. The MARR is usually a good value for this argument.

FV(A,i,N)

Returns the future worth at the end of period N for an annuity A, beginning at the end of period 1. The interest rate is i per period.

PV(A,i,N)

Returns the present worth of an annuity A, beginning at the end of period 1 and continuing until the end of period *N*. The interest rate is *i* per period. Note that this function works only with an annuity, while NPV works with any cash flow pattern.

Other Functions

There are many other specialized functions available, but the ones presented in this chapter and in Chapter 3 will solve all problems in the text. It is better to be intimately familiar with a few functions and use them well, rather than to be vaguely familiar with a many functions and use them poorly.

Cumulative Sum

To determine a cumulative sum, enter the formula SUM($base..base) into the first cell (base) of the range. The absolute addressing is needed for the first cell. As this formula is copied, the first cell remains fixed, and the last cell changes. This provides the cumulative sum up to and including the current location. See Template 4.5 for an example.

Template 4.1: Financial Measures of Merit

This template presents two approaches for determining equivalent worth measures. Since both approaches result in the same answer, the choice is a matter of preference.

In Approach A, the first order of business is to determine the PW of the cash flow. Note that the cash flow at period $k = 0$ must be added outside of the NPV function. Once the PW is known, the value is referenced in the AW calculation in cell C17 and in the FW calculation in cell C18.

Approach B calculates each measure of merit directly without specifically referring to the PW. Note that the PW in cell D16 is calculated using a different, but equivalent, formula than was used in cell C16.

An advantage of Approach A is that one can verify the intermediate calculation (PW). In Approach B, no intermediate results are available, making it difficult to validate the spreadsheet.

The template also includes the IRR in cell C19. Unlike the trial and error solution approach required when solving by hand, the IRR function is direct and simple. This is one reason for the increase of popularity of this metric.

	A	B	C	D	E	F	G
1	Financial Measures of Merit			**TEMPLATE 4.1**			
2							
3	MARR:	10%					
4							
5	Period	Net cash Flow					
6							
7	0	($10,000)					
8	1	$3,600					
9	2	$3,300					
10	3	$2,900					
11	4	$2,900					
12	5	$1,000					
13							
14			Approach	Approach			
15			A	B			
16	Present Value		$780.47	$780.47			
17	Annual Worth		($205.89)	$0.00			
18	Future Worth		$1,256.96	($0.50)			
19	Internal Rate of Return		13.48%				
20							
21							
22							
23	KEY:						
24		= User input					
25		=Unique formula					
26							
27	Uniqe Formulas						
28	C16	NPV(B3,B8:B12)+B7					
29	C17	PMT(B3,A12,C16)					
30	C18	FV(B3,A12,C17')					
31	C19	IRR(B7:B12,B3)					
32	D16	NPV(B3,B8:B12)*(1+B3)					
33	D17	PMT(NPV(B3,B8:B12)+B7,B3,A12)					
34	D18	FV(PMT(NPV(B3,B8:B12)+B3,B7,A12),B3,A12)					
35							
36							
37							
38							
39							
40							
41							

Template 4.2: Capital Recovery Amount

Calculating capital recovery amounts separately for spreadsheet problems is generally not necessary. However, students will find this template helpful for verifying homework problems and practicing for exams.

Two equivalent formulas for determining the capital recovery amount appear in cells B13 and B15. Formula B13 determines the PW of the cash flow range and then computes an annuity of that present amount. Formula B15 uses one of the closed form equations provided in the tenth edition of *Engineering Economy*.

	A	B	C	D	E	F
1	Capital Recovery Amount			**TEMPLATE 4.2**		
2						
3	MARR:	10%				
4						
5	Period	Cash Flow				
6	0	($10,000)				
7	1	$0				
8	2	$0				
9	3	$0				
10	4	$0				
11	5	$2,000				
12						
13	**CR (1):**	$0.00				
14						
15	**CR (2):**	($43,800.00)				
16						
17						
18						
19	KEY:					
20		= User input				
21		=Unique formula				
22						
23	Uniqe Formulas					
24	B13	PMT(NPV(B3,B7:B11)+B3,B6,A11)				
25	B15	(-B6-B11)PMT(B3,1,A11)+B11*B3				
26						
27						
28						
29						
30						

Template 4.3: Graphical Determination of IRR

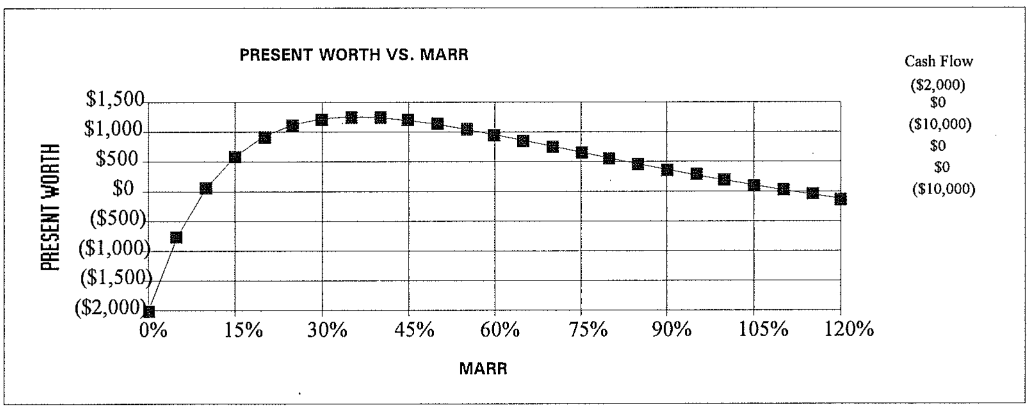
Template 4.3 provides a great deal of insight into the concept of IRR. The IRR is defined as the value of *i'* that makes the PW (or other equivalent worth measure) equal to zero. By plotting the PW as a function of *i'*, it is easy to determine the IRR graphically.

Varying the values in the cash flow range B6..B11 and observing the impact on the curve is a good way to get a feel for the IRR. Figure 4.2 illustrates the case of multiple IRRs. The Solver menu command can be used to determine the IRR by specifying the following in the Solver Dialog Box:

Set Cell : B14
To Maximize
By Adjusting Cell: A14
Constraint Cells: A14>0 (can go in any empty cell)
B14 = 0 (can go in any empty cell)
Number of Answers: 3

The results for the bottom cash flow in Figure 4.2 are 20%, 50%, and 100%. Solver is more appropriate than Goal Seeker, since multiple answers are present.

As a final exercise, observe what happens to the original curve by taking the opposite point of view (reverse the signs in the cash flow range). Note how this changes the algorithm for bracketing the IRR manually.



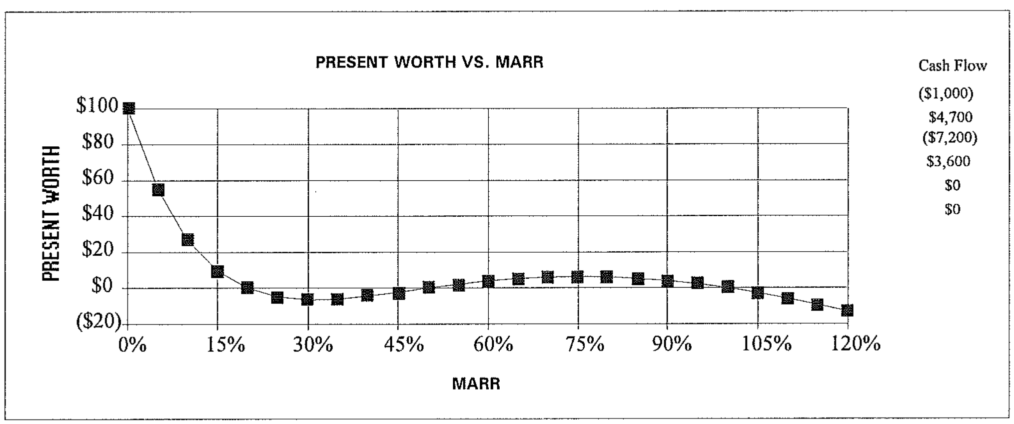
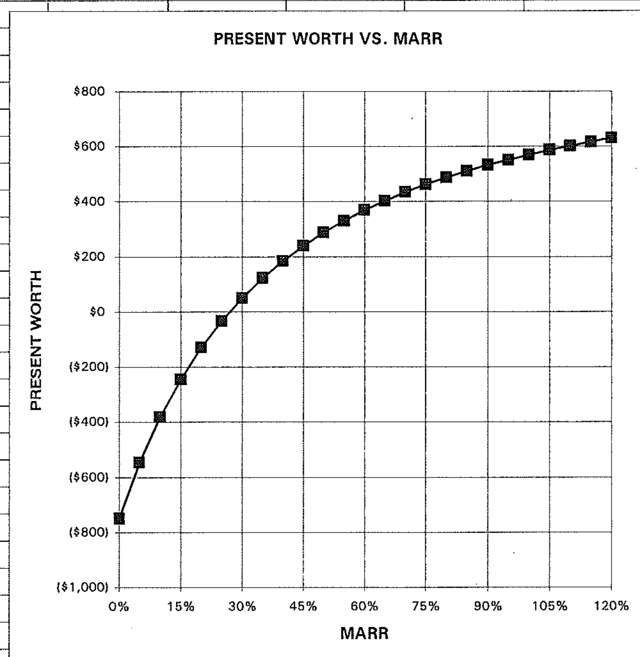


Figure 4.2 Multiple Rates of Return From Template 4.3

	A	B	C	D	E	F	G	H	I
1	GRAPHICAL DETERMINATION OF IRR					TEMPLATE 4.3			
2									
3	MARR:	10%		IRR:	27%				
4									
5	Year	Cash Flow							
6	0	$1,000		KEY:					
7	1	($500)			=User Input				
8	2	($500)			=Unique Formula				
9	3	($250)							
10	4	($250)		Unique Formulas					
11	5	($250)		E3	IRR(B6...B11)				
12				B14	NPV(A14, $B$7...$B$11)+$B$6				
13	i	NPV (i)							
14	0%	($750)							
15	5%	($547)							
16	10%	($382)							
17	15%	($244)							
18	20%	($130)							
19	25%	($32)							
20	30%	$51							
21	35%	$123							
22	40%	$185							
23	45%	$240							
24	50%	$288							
25	55%	$331							
26	60%	$369							
27	65%	$403							
28	70%	$434							
29	75%	$462							
30	80%	$488							
31	85%	$511							
32	90%	$533							
33	95%	$552							
34	100%	$570							
35	105%	$587							
36	110%	$603							
37	115%	$617							
38	120%	$630							
39									
40									



Template 4.4: External Rate of Return

This template illustrates the problem formulation approach when one of the standard '=' functions will not give the desired measure of merit. This approach can be adopted for other situations.

The ERR eliminates the need for a trial and error approach to determine a rate of return and eliminates the problem of multiple rates of return. Since there is no =ERR function, it is necessary to modify the cash flows so that existing functions can be utilized.

The first step is to break the cash flow into positive and negative components. This can be done manually by examining each value and placing it in the negative or positive columns (D and E). Two =IF functions serve the same purpose. Cell D8 will return only negative values or zero, while cell E8 returns only positive values or zero.

The next step is to determine the PW of all negative cash flows and FW of all positive cash flows. These calculations take place in cells D15 and E15. Cells F8 and F13 simply copy these results to the appropriate time period. Note that the periods 1, 2, ..., N-1 are blank, signifying no cash flow during those periods. Applying the =IRR function to the range F8..F13 calculates the IRR. Because the equivalent worth of all negative cash flows appears at year $k = 0$ and the equivalent worth of all positive cash flows appears at $k = N$, the result is actually the ERR. The value of the ERR appears in cell D15.

	A	B	C	D	E	F
1	External Rate of Return Method				**Template 4.4**	
2						
3	MARR	10%				
4	e:	8%				
5				**Cashflow breakdown for ERR**		
6	Period	Net Cash Flow		**Negative**	**Positive**	**Final**
7						
8	0	($10,000)		($10,000)	$0	($10,000)
9	1	$3,600		$0	$3,600	
10	2	$3,300		$0	$3,300	
11	3	$2,900		$0	$2,900	
12	4	$2,900		$0	$2,900	
13	5	$1,000		$0	$1,000	$16,569
14						
15				($10,000)	$16,569	65.69%
16				(NPV)	(FV)	ERR
17						
18	KEY:					
19		= User input				
20		= Unique formula				
21						
22	Unique Formulas					
23	D8	=IF(B8<0,b8,0)				
24	E8	=IF(B8>=0,B8,0)				
25	F8	=D15				
26	F13	=E15				
27	D15	=NPV(B4,D9:D13)+D8				
28	E15	=FV(B4,A13,PMT(B4,A13,(NPV(B4,E9:E13)+E8)))				
29	F16	=IRR(F8:F13,B3)				

Template 4.5: Payback

The payback method is frequently used in industry, yet it should be used with caution because it does not consider the time value of money. Many analysts use it as a supplemental measure of risk, in conjunction with one of the previous measures of merit.

This template demonstrates a cumulative sum. The formula in cell E10 uses absolute addressing to refer to the first value in the range and relative addressing to refer to the final cell in the range. As this formula is copied down the column, the last cell in the range changes, but the initial one remains the same. The result is the total from the first cell, up to and including the current cell.

Cell F10 uses an =IF function to return the value of φ (or φ'). The adjacent double quote symbols return a null character, leaving cells that do not satisfy the criteria blank. The first value to appear in column F is the payback period.

If cell B3 is 0, each value in the discount factor column C will be 1.0 and column F will return the regular (simple) payback period. Placing a value for the MARR into cell B# will return the discounted payback in column F.

	A	B	C	D	E	F	G
1	Payback Method			Template 4.5			
2							
3	MARR:	6.0%		NOTE:			
4				Regular payback: specify MARR = 0			
5				Discounted payback: specify MARR > 0			
6							
7			Discount	Adjusted	Cumulative	Payback	
8	Period	Cash Flow	Factor	Cash Flows	Cash Flow	Period	
9							
10	0	($5,000)	1.0000	($5,000)	($5,000)		
11	1	$1,000	0.9434	$943	($4,057)		
12	2	$2,000	0.8900	$1,780	($2,277)		
13	3	$1,000	0.8396	$840	($1,437)		
14	4	$1,000	0.7921	$792	($645)		
15	5	$2,000	0.7473	$1,495	$850	5	
16							
17							
18	KEY:						
19		= User input					
20		= Unique formula					
21							
22	Unique Formulas						
23	C10	=(1+$B$3)^-A10					
24	D10	=C10*B10					
25	E10	=SUM($D$10:$D10)					
26	F10	=IF(E10>=0,A10," ")					

Chapter 5
Comparison of Alternatives

Tenth Edition Highlights

Chapter 3 introduced the basic time value of money concepts. These fundamentals were synthesized into measures of merit for comparing alternatives with dissimilar cash flows in Chapter 4. Chapter 5 in the Tenth Edition of *Engineering Economy* (DeGarmo, Sullivan, Bontadelli, and Wicks) provides the rules for rationally selecting between alternatives. This trilogy completes the foundation of engineering economic analysis. The sequel addresses further model enhancements and special applications of this core material.

Two additional measures of merit are introduced in this chapter. The Capitalized Worth (CW) is an equivalent worth technique that transforms a perpetual series of uniform end-of-period cash flows to a single equivalent at year $k = 0$. The Incremental Internal Rate of Return (IIRR) is a rate of return method that addresses the problems and inconsistencies of the IRR method. It is identical to the IRR approach except for its use of incremental cash flows.

Every alternative under consideration must provide equivalent results. Functional equivalence is an engineering determination. Each alternative must also provide these results over the same time frame. Repeatability and the cotermination are two methods for providing equivalent time frames. Repeatability extends the study period to the least common multiple of each alternative under consideration. Since it assumes that the cash flows will be the same in subsequent lives, it cannot be used if inflation is present. Cotermination selects a shortened, common time frame for all alternatives. This approach requires an estimate of the market value for alternatives whose useful lives are greater than the study period. The imputed market value technique is frequently used to estimate the market value in these cases.

Alternatives under consideration may be: (1) independent, (2) mutually exclusive, or (3) contingent.

If investments are independent, selection of one alternative does not affect the selection of another, subject to running out of capital. One can compute an equivalent worth measure of merit, such as PW, and use this value to rank all alternatives. Alternatives are selected in descending order of PW until the capital budget is exhausted.

More commonly, alternatives are mutually exclusive. That is, one, and only one alternative will be selected. The decision rule is to select the alternative requiring the least capital investment, unless the incremental investment in another alternative is justified by its incremental savings.

The selection of contingent alternatives is conditional on the selection of one or more other alternatives. This analysis is complex, but can be simplified for small problems by enumerating every combination of alternatives. The resulting combinations are now mutually exclusive, and the same decision rules apply.

Formula Summary

Capitalized Worth

$$CW = \frac{A}{i}$$

Imputed Market Value

$$MV_{imputed} = CR_N * (P/A,i,N-k) + S(P/F,i,N-k)$$

Here, CR_N refers to the Capital Recovery cost for the full study period (N periods), k refers to the length of the shortened study period, and S is the salvage (market) value at the end of period N.

Overview

Template 5.1 illustrates how to incorporate repeatability into a model with simple copy and paste commands. When cotermination is used instead of repeatability, Template 5.2 uses the Backsolver menu command to determine imputed market value. Template 5.1 also illustrates the basics for incremental analysis. Capitalized worth is shown in template 5.3.

Templates 5.4 and 5.5 depart from the form used in prior templates. As the analyses become more involved, changing numbers on the template or copying formulas to larger ranges is no longer possible. One can still incorporate features of the prior templates, but now one must take time to formulate the model for each specific problem.

Template 5.4 illustrates the IIRR for four alternatives. Note that in formulating the problem, the user must first ensure that the alternatives are ranked in order of increasing investment. Next, the user must select the alternatives for incremental comparison in the correct sequence. Here, the model is trivial, knowing which alternatives to compare is paramount.

Template 5.5 introduces a new menu command and function. Matrix multiplication is used to reduce the number of formulas in the spreadsheet and new functions are used to identify solutions. As in the previous template, setting up the problem is the key to success. The Solver menu command can also be used for this type of problem. See the template in Chapter 12 for further information.

Spreadsheet Templates

5.1	repeat	Repeatability & Incremental analysis
5.2	imputemv	Imputed market value
5.3	capworth	Capitalized worth
5.4	iirr	Incremental internal rate of return
5.5	conting	Evaluation of contingent alternatives

New Features and Functions

ABS(·)

Returns the absolute value of the argument. The argument can be a value or a cell containing a value or formula.

EXACT(reference text,compared text)

Returns a logical true (1) if the reference text matches the compared text. Returns a logical false (0) if they do not match.

Matrix Multiplication — Range | Analyze | Multiply Matrix...

Multiplies two matrices (or vectors). The number of columns in the first matrix must equal the number of rows in the second matrix. These calculations are not automatically updated as the spreadsheet changes!

Template 5.1: Repeatability & Incremental Analysis

This spreadsheet illustrates repeatability and incremental analysis. The copy and paste functions, coupled with simple arithmetic operators, make both very easy to implement.

Alternative *A* has a useful life of 10 years, while Alternative *B's* useful life is 5 years. To compare these alternatives over an equivalent time period, we assume Alternative *B* is replaced at the end of period 5 with an exact equivalent, Alternative *B'*.

To create the cash flow for Alternative *B'*, copy the cash flow from C6..C11 and paste it to D11..D16. Note that period 5 is $k = 0$ for Alternative *B'*, so there is an overlap. This is why we did not paste this range directly in Column C. Column E adds C and D to form a composite alternative with a ten period life. This composite is now directly comparable with Alternative *A*.

Row 18 contains the PW calculation for each column and illustrates what can happen when alternatives are not compared over equivalent time horizons. Note that the PW of Alternative *B* is less negative than Alternative *A*, and therefore *B* would be selected. However, the composite Alternative *B* PW in Column E shows that Alternative *A* is the better choice.

Instead of calculating the PW of each alternative, one can calculate the PW of the difference in cash flows. The decision will be the same, provided the alternative with the lower investment is the base. Column G calculates the difference between Alternatives *A* and (Net) *B*. The PW of this incremental cash flow is positive, indicating that the extra investment in Alternative *B* is justified. This is the same conclusion as with the individual cash flows.

	A	B	C	D	E	F	G
1	Repeatability & Incremental Analysis				Template 5.1		
2							
3	MARR	10%					
4					Alternative B		Incremental
5	Period	Alternative A	Alternative B	Alternative B'	Net		Cash Flow
6	0	($40,000)	($20,000)		($20,000)		($20,000)
7	1	$6,000	$4,500		$4,500		$1,500
8	2	$6,000	$4,500		$4,500		$1,500
9	3	$6,000	$4,500		$4,500		$1,500
10	4	$6,000	$4,500		$4,500		$1,500
11	5	$6,000	$4,500	($20,000)	($15,500)		$21,500
12	6	$6,000		$4,500	$4,500		$1,500
13	7	$6,000		$4,500	$4,500		$1,500
14	8	$6,000		$4,500	$4,500		$1,500
15	9	$6,000		$4,500	$4,500		$1,500
16	10	$6,000		$4,500	$4,500		$1,500
17							
18	PW:	($3,132.60)	($2,941.46)		($4,767.87)		$1,635.28
19							
20							
21	KEY:			Unique Formulas			
22		=User Input		E6	= SUM(C6..D6)		
23		=Unique Formula		G6	=B6-E6		
24				B18	= NPV($B$3,B7.B16)+B6		
25							

Template 5.2 Imputed Market Value

If the study period is less than the useful life (N) of an asset, one must determine a market value at the end of the study period. The imputed market value technique determines this value such that the Capital Recovery amount for N periods is the same as the Capital Recovery amount for k periods.

The Goal Seek menu command is used instead of a dedicated formula to calculate the imputed market value in cell C11. Cells B16 and C16 determine the Annual Worth over the full and reduced study periods, respectively. Cell D16 calculates the difference between these two cells.

To calculate the imputed market value, enter the following values into the Goal Seek dialog box:

Make Cell:	D16
Equal to value:	0
By changing cell(s):	C11

The imputed market value can be determined for any other study period length by changing the "Make cell" location.

	A	B	C	D	E	F	G
1	Imputed Market Value				Template 5.2		
2							
3	MARR	10%					
4					Alternative B		Incremental
5	Period	Alternative A	Alternative B	Alternative B'	Net		Cash Flow
6	0	($40,000)	($20,000)		($20,000)		($20,00
7	1	$6,000	$4,500		$4,500		$1,50
8	2	$6,000	$4,500		$4,500		$1,50
9	3	$6,000	$4,500		$4,500		$1,50
10	4	$6,000	$4,500		$4,500		$1,50
11	5	$6,000	$0	($20,000)	($20,000)		$26,00
12	6	$6,000		$4,500	$4,500		$1,50
13	7	$6,000		$4,500	$4,500		$1,50
14	8	$6,000		$4,500	$4,500		$1,50
15	9	$6,000		$4,500	$4,500		$1,50
16	10	$6,000		$0	$0		$6,00
17							
18	PW:	($3,132.60)	($5,735.61)		($9,296.97)		$6,164.3
19							
20							
21	KEY:			Unique Formulas			
22		=User Input		E6	=SUM(C6..D6)		
23		=Unique Formula		G6	=B6-E6		
24				B18	=NPV($B$3,B7.B16)+B6		

Template 5.3: Capitalized Worth (CW)

Capitalized Worth is an appropriate measure whenever perpetual cash flows are involved. Two hundred periods are sufficient for the spreadsheet to return results that are equivalent to "forever" for most commonly encountered interest rates. The most direct method to determine the CW is to enumerate the cash flows for 200 periods and then determine the equivalent AW of the range.

A less space intensive approach is to determine the cycle of repeated cash flows. In this template, a single investment at year $k = 0$ is followed by one set of annual expenses and one set of expenses that occur every four years. Thus, the cycle for this study is 4 years.

Column E adds the various expenses in columns B, C, and D to obtain the net cash flow. The Capitalized Worth of the perpetual annuity appears in cellE13 and capitalized worth of the entire project appears in cell E14.

Note that by excluding the investment at year $k = 0$ from the PW calculation in cell E13, we get the PW of the Annuity alone.

	A	B	C	D	E	F
1	Capitalized Worth				Template 5.3	
2						
3	MARR:	8%				
4	Cycle:	4				
5						
6	Period	Investment	Annual Expenses	Equipment Replacement	Net Annual Requirements	
7	0	($100,000)			($100,000)	
8	1		($30,000)		($30,000)	
9	2		($30,000)		($30,000)	
10	3		($30,000)		($30,000)	
11	4		($30,000)	($20,000)	($50,000)	
12						
13				PW of annuity:	($430,480.20)	
14				CW:	($530,480.20)	
15						
16	KEY:					
17		= User input				
18		= Unique formula				
19						
20	Unique Formulas					
21	E7	=SUM(B7:D7)				
22	E13	=PMT(B3,B4,NPV(B3,E8:E11))/B3*(-1)				
23	E14	=B7+E13				
24						
25						

Template 5.4: Incremental Internal Rate of Return (IRR)

As was the case with the ERR, there is no =IRR function. However, after calculating the incremental cash flow between properly ranked alternatives, the =IRR function can be applied to the incremental cash flow to obtain the IRR.

There are two advisories to this template. First, ensure that the alternatives are ranked from least to greatest investment. Second, when computing differences, make sure that the incremental cash flows are negative at year k = 0.

Row 14 determines the IRR of all alternatives. Since IRR_{Alpha} < MARR, Alpha is excluded from further consideration. Beta requires the second smallest investment and since IRR_{Beta} > MARR, it becomes the default alternative. It is not necessary to use IRR as the measure of merit to determine initial feasibility. Any equivalent worth method could have been used as well.

The first comparison is between Beta and Gamma, since Gamma requires the next highest investment. The incremental cash flows appear in B18..B23. Since $IRR_{Beta\text{-}Gamma}$ < MARR, the incremental investment for Gamma is not justified. Gamma is eliminated from further consideration and Beta remains the default alternative.

Beta is next compared to the most expensive investment, Delta. The incremental cash flow appears in C18..C23. Since $IRR_{Beta\text{-}Delta}$ > MARR, the extra investment is justified, and Delta is selected as the new default. Since there are no other alternatives, Delta is selected.

	A	B	C	D	E	F
1	Incremental Internal Rate of Return				**Template 5.4**	
2						
3	MARR:	10%				
4						
5	Alternatives Under Consideration					
6	**Period**	**Alpha**	**Beta**	**Gamma**	**Delta**	
7	0	($10,000)	($11,000)	($13,000)	($16,000)	
8	1	$2,500	$3,500	$3,800	$5,000	
9	2	$2,500	$3,500	$3,800	$5,000	
10	3	$2,500	$3,500	$3,800	$5,000	
11	4	$2,500	$3,500	$3,800	$5,000	
12	5	$2,500	$3,500	$3,800	$5,000	
13						
14	IRR:	7.9%	17.8%	14.1%	17.0%	
15						
16	Incremental Analysis					
17		**Beta:Gamma**	**Beta:Delta**			
18	0	($2,000)	($5,000)			
19	1	$300	$1,500			
20	2	$300	$1,500			
21	3	$300	$1,500			
22	4	$300	$1,500			
23	5	$300	$1,500			
24						
25	IIRR	-8.9%	15.2%			
26						
27	KEY:					
28		= User input				
29		= Unique formula				
30						
31	Unique Formulas					
32	B14	=IRR(B7:B12,$B$3)				
33	B18	=D7-C7				
34	C18	=E7-C7				
35	B25	=IRR(B18:B23,$B$3)				
36						

Template 5.5: Evaluation of Contingent Alternatives

Enumerating every possible combination of contingent alternatives to form a new set of mutually exclusive alternatives can become burdensome for even a moderate number of alternatives. This template reduces the number of formulas required to select the combination of contingent alternatives that maximizes PW subject to the capital constraints.

There are three user-input sections. The MARR and maximum capital for investment are in B3: B4. The alternatives and their cash flows appear in range A6:F11. Finally, the mutually exclusive combinations of alternatives appear in the range A14: F22. This final input area is unique for each problem and is most prone to error. The sequence of 0s and 1s is determined from the problem statement.

Despite the complexity of this problem, there are few formulas. The PW of each alternative is calculated in G7: G12. Determining the PW and the total investment of each mutually exclusive combination can be done with formulas but is tedious. A faster approach is to use matrix multiplication. To determine total investment, the Multiply Matrix menu command dialog box should contain the following values:

First matrix:	B17: F22
Second matrix:	B7: B11
Resulting matrix	G17

To determine PW the Multiply Matrix dialog box should contain the following values:

First matrix:	B17: F22
Second matrix:	B7: B11
Resulting matrix	G17

The formula in I17 uses an IF function to determine if the mutually exclusive combination investment is under the capital budget. If it is the label "YES" appears in this column.

Cell K17 is an intermediate step. The "hidden" format is used so it is not displayed. If column I contain any other text, a null character is placed in column K.

Cell J17 identifies the maximum value in column K. Since only feasibly combinations have PWs in this column; the maximum value is he best mutually exclusive combination.

Mutually exclusive combination 5 is identified as best. By looking at the 0/1 matrix, we see that alternatives B2 and C2 are selected.

Matrix calculations are NOT automatically updated when the spreadsheet is changed. If any changes are made to the user-input regions, both matrix multiplication operations must be reported.

	A	B	C	D	E	F	G	H	I	J
1	Contingent Selection			**Template 5.5**						
2										
3	MARR:	10.00%								
4	Max Invest	$48,000								
5				Period						
6	Alternative	0	1	2	3	4	PW			
7	B1	($50,000)	$20,000	$20,000	$20,000	$20,000	13,397			
8	B2	($30,000)	$12,000	$12,000	$12,000	$12,000	$8,038.39			
9	C1	($14,000)	$4,000	$4,000	$4,000	$4,000	(1,321)			
10	C2	($15,000)	$5,000	$5,000	$5,000	$5,000	$849.33			
11	D	($10,000)	$6,000	$6,000	$6,000	$6,000	$9,019.19			
12										
13				**Alternative**						
14	**Mutually**						**Total**	**PW**	**Feasible?**	**Selection**
15	**Exclusive**	**B1**	**B2**	**C1**	**C2**	**D**	**Investment**			
16	**Combination**									
17	1	0	0	0	0	0	$0	$0	YES	
18	2	1	0	0	0	0	($50,000)	$13,397	no	
19	3	0	1	0	0	0	($30,000)	$8,038	YES	
20	4	0	1	1	0	0	($44,000)	$6,718	YES	
21	5	0	1	0	1	0	($45,000)	$8,888	YES	BEST
22	6	0	1	1	0	1	($54,000)	$15,737	no	
23										
24										
25	KEY:									
26		=User Input								
27		=Unique Formula								
28										
29	Unique Formulas									
30	G7	NPV($B$3,C7:F7)+B7		IF(ABS(G17)>$B$4,"no","YES")						
31	I17	IF(ABS(G17)>$B$4,"no","YES")								
32	J17	IF(K17=MAX($K$17:$K$22),"BEST","")								
33	K17	IF(EXACT("YES",I17),+H17,"")								
34										
35										
36										
37										
38										
39										
40										
41										
42										
43										
44										
45										
46										

Chapter 6
Evaluating Projects with the Benefit/Cost Ratio Method

Tenth Edition Highlights

The public sector involves all branches and levels of government. The Benefit/Cost Ratio is introduced as a new measure of merit to reflect the differences between the public and private sectors. These ratios consider the equivalent benefits and the equivalent costs of an alternative. If the ratio is greater or equal to one, the alternative is acceptable. By convention, the signs of the cash flows are all taken to be positive in B/C analysis. Although not commonly encountered, B/C ratios can be used in the analysis of private sector projects.

There is no unique Benefit/Cost ratio formulation. The Tenth Edition of *Engineering Economy* (DeGarmo, Sullivan, Bontadelli, and Wicks) presents the Conventional B/C and the Modified B/C ratios based on both AW and PW measures of merit. The value of B/C ratios calculated using AW are identical to those calculated with PW. Since most long-term project estimates are expressed in annual costs, the AW formulation is usually more convenient.

The values of the Modified and Conventional B/C ratios are not identical. However, the decision regarding an alternative's acceptability is consistent between the two methods.

Mutually exclusive alternatives are evaluated in a similar fashion to the IIRR method: by incremental analysis. Alternatives are ranked in increasing order of equivalent costs, with the lowest cost alternative serving as the initial default. If the incremental B/C ratio is >1, the higher cost alternative is preferred.

Notation Summary

AW(B):	Annual worth of benefits
AW(D):	Annual worth of disbenefits
AW(O&M):	Annual worth of operating and maintenance costs
B/C:	Benefit/Cost ratio. Several formulations are available
CR:	Capital Recovery amount
I:	Investment
PW(B):	Present worth of benefits
PW(D):	Present worth of disbenefits
PW(O&M):	Present worth of operating and maintenance costs
PW(S):	Present worth of market (salvage) value

Formula Summary

The formulas in the text can be consolidated to the four general purpose formulas in Table 6.1. If market value or disbenefits are not applicable, simply omit those terms in the calculations.

Table 6.1. Consolidated B/C ratio formulas.

Type of B/C Ratio	Annual Worth	Present Worth
Conventional	$B/C = \dfrac{AW(B) - AW(D)}{CR + AW(O \& M)}$	$B/C = \dfrac{PW(B) - PW(D)}{I - PW(S) + PW(O \& M)}$
Modified	$B/C = \dfrac{AW(B) - AW(O \& M) - AW(D)}{CR}$	$B/C = \dfrac{PW(B) - PW(O \& M) - PW(D)}{I - PW(S)}$

Overview

The various B/C ratio formulations presented in the text give different numeric values, just as AW, PW and FW do for the same alternative. As with the equivalent worth measures, all B/C ratios provide consistent recommendations, despite their different values. The issue is not which formulation to select, but applying the same formulation consistently throughout the analysis.

Template 6.1 calculates the conventional and modified B/C ratios using both AW and PW. Template 6.2 shows how to use B/C ratios to select between mutually exclusive alternatives. Both templates demonstrate that the AW and PW formulations are equivalent.

Spreadsheet Templates

6.1	bcratios	Benefit/cost ratios
6.2	bcselect	Mutually exclusive selection using Conventional Benefit/cost ratio

Template 6.1: Benefit/Cost Ratios

This template provides formulas for conventional and modified B/C ratios, using both AW and PW. The results in B17..C17 and B18..C18 confirm that both the AW and PW formulation yield identical answers for each type of ratio.

To aid in verifying the model, the CR amount is calculated separately in cell B14. CR is needed in the PW formulation, but not in the AW formulation.

If disbenefits or market value are not considered, enter "0" in the corresponding input cell. To treat disbenefits as a reduction in benefits, enter 0 in cell B12 and the amount "benefits - disbenefits" in cell B11.

Note that the user input values (other than investment) are on an annual basis.

	A	B	C	D	E
1	**B/C Ratios**			**Template 6.1**	
2					
3	MARR:	10%			
4	Life:	20			
5					
6	NOTE: all values should be positive				
7					
8	Investment:	$1,200,000.00			
9	Market Value:	$0.00			
10	Annual O&M:	$197,500.00			
11	Annual Benefits:	$490,000.00			
12	Annual Disbenefits:	$100,000.00			
13					
14	CR:	$140,951.55			
15					
16	**B/C Ratio:**	**AW**	**PW**		
17	**Conventional**	1.15	1.15		
18	**Modified**	1.37	1.37		
19					
20	Key:				
21		=User Input			
22		=Unique Formula			
23					
24	Unique Formulas				
25	B14	=PMT(B3,B4,-B8)-PMT(B9/(1+B3)^B3,B4,B3)			
26	B17	=(B11-B12)/(B14+B10)			
27	C17	=PV(B3,B4,B11-B12)/(-B8-B9+PV(B3,B4,B10))			
28	B18	=(B11-B10-B12)/B14			
29	C18	=PV(B3,B4,B11-B10-B12)/(-B8-B9/(1+B3)^B4)			
30					

Template 6.2: Mutually Exclusive Selection Using Conventional Benefit/Cost Ratios

Three alternative routes, A, B, and C have construction costs, annual maintenance costs, annual savings due to fire prevention, annual benefits for recreation, and annual benefits for timber. Each has an expected life of 50 years. The Conventional B/C ratio is used throughout this template.

The PW summary of benefits and costs for each alternative are in the range B14: C16. The AW summary of the benefits and costs appear in the range E14: F16.

Since the alternatives are mutually exclusive they must be ranked from smallest to largest denominator for incremental analysis. Thus, A becomes the default, if one alternative must be selected. It is compared next to alternative B. The better alternative of this pair will then be compared with Alternative C.

The incremental cost from the do nothing alternative to Route A is shown on line 21. Since the B/C ration is <1, the do nothing alternative is preferred. However, since one route must be selected, it would be A, since it incurs the least cost.

Route A is next compared to route B. The incremental benefits do not exceed the incremental cost, leaving A as the choice. Finally, route A is compared to route C. Again the route is not justified and A remains the recommendation.

The same analysis is repeated using the Annual worth formulation in the range A27: D29. Note that the incremental B/C ratios are identical for both formulations, and that alternative A is selected by both methods.

1	Selection Using the Conventional Benefit/Cost Ratio			**Template 6.2**		
2						
3	Life:	50				
4	MARR:	8%				
5	**Route**	**Construction**	**Ann. Maint**	**Annual**	**Annual**	**Annual**
6		**Cost**	**Cost**	**Savings Fire**	**Benefit**	**Benefit**
7					**Recreation**	**Timber**
8	A	$185,000.00	$2,000	$8,000	$3,000	$500
9	B	$220,000	$3,000	$7,000	$6,500	$1,500
10	C	$290,000	$4,000	$12,000	$6,000	$2,800
11						
12		**Present Worth Formulation**		**Annual Worth Formulation**		
13	**Route**	Benifts	Costs		Benifts	Costs
14	A	-$140,685	$160,533		$11,500	-$13,122
15	B	-$183,502	$183,300		$15,000	-$14,983
16	C	-$254,456	$241,066		$20,800	-$19,705
17	Rank Alternatives by increasing denominator:			A=default, then B, then C		
18						
19	Present Worth Formulation					
20		Inc. Benefit	Inc. Cost	Inc. B/C		
21	0-A	-$140,685	$160,533	-0.876362159		
22	A-B	-$42,817	$22,767	-1.88070926		
23	A-C	-$70,954	$57,767	-1.228293078		
24						
25	Annual Worth Formulation					
26		Inc. Benefit	Inc. Cost	Inc. B/C		
27	0-A	$11,500	-$13,122	-0.876362159		
28	A-B	$3,500	-$1,861	-1.88070926		
29	A-C	$5,800	-$4,722	-1.228293078		
30						
31	Recommendation (and B?C values) is the same for both methods.					
32	Select route A, given that one must be selected.					
33						
34	KEY:					
35		=User input				
36		=Unique formula				
37						
38	Unique Formulas					
39	B14	PV($B$4,$B$3,SUM(D8:F8))				
40	C14	PV($B$4,$B$3,C8)+B8				
41	E13	SUM(D8:F8)				
42	F14	PMT($B$4,$B$3,B8)+C8				
43	B21	B14				
44	C21	C14				
45	D21	B21/C21				
46	B22	B14-B13				
47	C22	C14-C13				
48	B23	B15-B13				
49	C23	C15-C13				
50	B27	E13				
51	C27	F13				
52	D27	B27/C27				
53	B28	E14-E13				
54	C28	F14-F13				
55	B29	E15-E13				
56	C29	F15-F13				
57						

Chapter 7
Depreciation and Income Taxes

Tenth Edition Highlights

Accurate decisions require a model that adequately portrays the real system. Chapters 1 through 6 develop the basic time value of money model and decision criteria. Chapters 7, 8, 9, 10, and 14 provide refinements that incorporate more real world behavior to our financial models.

Chapter 7 presents the basic income tax and depreciation mechanisms. While the coverage is adequate for project evaluation, it is not intended to be a full presentation of current Internal Revenue Service codes. However, the treatment is sufficient to illustrate how depreciation affects income taxes, and how income taxes affect project cash flows. Our purpose is to select between alternatives, not to file income tax returns!

Depreciation is an accounting construct representing the decrease in value of an asset, typically over time. It is a non-cash cost and a sunk cost, but not a cash flow. Although we do not consider sunk costs in our evaluations, depreciation is an exception because of its impact on income taxes. Depreciation lowers taxable income, and thus the income tax paid, which is a cash flow.

Income taxes are major expenses and deserve explicit consideration in our analyses. All other forms of taxes are aggregated and included with other expenses. Current tax codes specify MACRS depreciation. Straight line, sum-of-the-year's digits, and declining balance are historical depreciation methods. Straight line and declining balance depreciation are used to determine the MACRS recovery rates. The historical depreciation methods may also come into play during replacement analysis.

Depletion applies to natural resources. It represents the decrease in value for a piece of property as the natural resources are removed.

The tabular format in Figure 7-5 in the Tenth Edition of *Engineering Economy* (DeGarmo, Sullivan, Bontadelli, and Wicks) provides a convenient structure for transforming before-tax cash flows into after-tax cash flows. Observing the sign convention when using the associated formulas is important. The time value of money is not considered within the table.

The table has three distinct periods: (1) $k = 0$, representing the purchase of the asset, (2) $k = 1, 2, \ldots, N$, corresponding to the useful life of the asset, and (3) $k = N$, representing the disposal of the asset. Period N appears in the table twice, since different rules may apply to income derived from the sale of a capital asset.

Depreciation and taxation concepts are new to many engineering students. Thus, it is not surprising that most focus intently on the calculations and consider the problem complete once the ATCF table is complete. However, the goal of engineering economic analysis is to select between competing alternatives. The depreciation and tax calculations are necessary to determine the ATCF, which is then used to determine the measure of merit of each alternative. Thus, the ATCF column serves as the input for our time value of money models and the actual decision process.

Notation Summary

ATCF	After-tax cash flow
B	Adjusted cost basis
BTCF	Before-tax cash flow
BV_k	Book value at end of period k
d_k	Depreciation in year k
d_k^*	Cumulative depreciation through end of year k
E_k	Cash outflows during period k
G_N	Gain (loss) on disposal at end of period N
MV	Market value

N	Depreciable life of asset (years)
NIBT	Net income before income taxes
NIAT	Net income after income taxes
R	Ratio of depreciation to book value for declining balance method
R_k	Positive cash inflow during period k
r_k	MACRS recovery rate for year k
SV_N	Estimated market (salvage) value at end of year N
t	Effective income tax rate for ordinary income
T_k	Income taxes paid during year k

Formula Summary

Depreciation Method	d_k	d_k^*	BV_k
SL	$(B - SV_N) / N$	$k * d_k$ for $1 \le k \le N$	$B - d_k^*$
DB	$B * (1 - R)^{k-1} * (R)$	$B * [1 - (1 - R)^k]$	$B * (1 - R)^k$
SYD	$(B - SV_N) * [2 (N - K + 1) / \{N (N + 1)\}]$	$B - BV_k$	$B - [2 (B - SV_N) / N] k + [(B - SV_N) / \{N (N + 1)\}] k (k + 1)$
MACRS	$r_k * B$ $1 \le k \le N + 1$	•	

Declining Balance Depreciation Method

R = depreciation percent / N $(0 \le R \le 1)$

Units-of-Production Depreciation Method

Depreciation per unit = $(B - SV_N)$ / (Estimated lifetime production count)

Income Tax Formulas

Before-tax MARR ≈ After-tax MARR / (1 - effective income tax rate)

Taxable income = gross income - all expenses except capital expenditures - depreciation and other deductions

t = state rate + federal rate * (1 - state rate)

$G_N = MV_N - BV_N$

$BTCF_k = R_k - E_k$

$T_k = -t\ (BTCF - d_k)$

$NIAT_k = (BTCF - d_k)\ (1 - t)$

$ATCF_k = BTCF_k + T_k$

After-tax table (see Figure 7-5)

Year	[A] BTCF	[B] Depreciation	[C] Taxable Income	[D] Cash flow for Income taxes	[E] ATCF
k	$R_k - E_k$	d_k	= [A] - [B]	= -t [C]	= [A] + [D]

Overview

Spreadsheets simplify the process of transforming BTCFs into ATCFs and ultimately determining the after-tax measure of merit. Beside the obvious advantage of performing the numeric calculations, they provide structure for problem formulation. This structure reduces much of the confusion that confronts students who are new to depreciation and income tax concepts.

Several depreciation methods are included in the financial functions. However, using a tabular approach for determining d_k, d_k^* and BV_k helps clarify the depreciation basics. Once these are understood, the shorter functions are more convenient. Also, be aware that not all functions behave as their name suggests. It is good practice always to verify that the functions return the results that you expect!

Template 7.1 illustrates the historic depreciation methods using both the tabular format and the functions. Template 7.2 uses a lookup function to return the appropriate MACRS recovery rates value from a table of values. Template 7.3 shows how to determine the MACRS recovery rates. The tabular approach shows the switching mechanism from DB to SL, while the VDB function would be used in actual practice. Template 7.4 converts BTCFs to ATCFs using the format of Figure 7-5 in the Tenth Edition of *Engineering Economy*.

New Features and Functions

SLN(B,SV_N,N)

Returns d_k based on straight line depreciation. Since $d_1 = d_2 = ..., = d_N$, specifying the year with this function is not necessary.

SYD(B,SV_N,N,k)

Returns d_k based on the sum-of-the year's digits method.

DDB(B,SV_N,N,k)

Returns d_k based on the double declining (200%) method. This function should be used with caution, since it may not fully depreciate an asset with a low market (salvage) value. There are also no provisions for switching to straight line depreciation. VDB is a better choice.

VDB(B,SV_N,N, start, end,percent, switch)

Returns d_k for the general declining balance method. The *percent* argument is 2 for double declining balance, 1.5 for 150 % declining balance, etc. The *start* and *end* arguments permit use of the half year (and half month) conventions. To permit switching to the straight line method, set the *switch* argument to 0. A value of 1 for this argument disallows switching. This is the most general declining balance function, and is recommended over all others.

HLOOKUP(class life,table range,k)

This function returns a value from a specific location in a table. The location is specified by values in the header row and the number of rows below the header. In this specific application, the function returns the MACRS recovery rate for a specific class life and year. The class life appears in the header row. The year labels appear along the left of the table, but are NOT part of the table range.

MAX(range)

Returns the maximum value in a range.

MIN(range)

Returns the minimum value in a range.

Template 7.1: Historic Depreciation Methods

This template calculates d_k, d_k*, and BV_k for straight line, sum-of-the-year's digits, and declining balance depreciation using common values of cost basis, market value, and life entered in cells B3..B5. The tabular format and corresponding = function appear in each section.

A tabular format requires more space than the = functions, but it provides more insight into how d_k, d_k*, and BV_k, are determined. Once the depreciation methods are understood, one can switch to the less cumbersome = functions.

The straight line section allocates the difference between the cost basis and market value equally over the life of the asset. A cumulative sum is used to determine d_k*. The = SLN function determines d_k; however, one must still resort to the tabular approach to determine d_k*, and BV_k.

Column B in the sum-of-the-year's digits section can be consolidated directly into the "ratio" column by using the formulas given in the text. The approach illustrated in Template 7.1 provides a better feel for how this method works. Like the = SLN function, the = SYD function provides results that are identical to the tabular format.

The declining balance with the switch to straight line is the most complicated method in the template. Recall that since the market value does not enter the calculations, the value of BV_n will not equal the market value. Depreciation switches from DB to SL in the year that SL provides a greater deduction.

Column B computes d_k based on the declining balance method, where the percent DB is specified in cell B31. Column C computes d_k based on straight line depreciation, without considering the market value. The maximum value of d_k is selected in Column D and is used to determine the book value in Column E. An = if function in Column F indicates which method (SL or DB) provides the maximum value of d_k.

The remaining columns ensure that the book value does not drop below the market value. Column G compares the book value to the market value. If BV<SV, it returns the label "adj", showing that an adjustment is required. Column H uses logical operators to determine if the current and previous year depreciation are both SL. If so, the value **1** is returned, indicating that d_k should not change. Column J is hidden and computes the SL depreciation for all years with the label "adj" in column G. The check in column H is used to ignore all but the first SL value. Column I gives the d_k value for the declining balance method with a switch to straight line.

The DDB = function is in column L. Note that although the = function drastically reduces the number of formulas, it poses a different problem. When the function switches to straight line, it depreciates the full amount remaining in the year of the switch, rather than over the remaining life, as is done in the Tenth Edition of *Engineering Economy*. The tabular format follows the text approach.

The = DDB function results drive home a key point: make sure that you fully understand how all = functions work before using them. It is good practice to perform the calculations " long hand" with the spreadsheet, and then compare these results with those of the = function.

	A	B	C	D	E	F	G	H	I	J	K	L
1	Depreciation Methods Used Prior to 1981				Template 7.1							
2												
3	Purchase:	$5,000										
4	Salvage:	$1,000										
5	Life:	5										
6												
7												
8	**Straight Line Method**					**d(k)**						
9	**Year**	**d(k)**	**d*(k)**	**BV**		**= function**						
10	**0**			$5,000								
11	**1**	$800	$800	$4,200		$800						
12	**2**	$800	$1,600	$3,400		$800						
13	**3**	$800	$2,400	$2,600		$800						
14	**4**	$800	$3,200	$1,800		$800						
15	**5**	$800	$4,000	$1,000		$800						
16												
17												
18	**Sum-of-the-Year's Digits Method**											
19		**Reverse**						**d(k)**				
20	**Year**	**Digits**	**Ratio**	**d(k)**	**d*(k)**	**BV**		**= function**				
21	0					$5,000						
22	1	5	33.33%	$1,333	$1,333	$3,667		$1,333				
23	2	4	26.67%	$1,067	$2,400	$2,600		$1,067				
24	3	3	20.00%	$800	$3,200	$1,800		$800				
25	4	2	13.33%	$533	$3,733	$1,267		$533				
26	5	1	6.67%	$267	$4,000	$1,000		$267				
27	Total:	15										
28												
29												
30	**Declining Balance with Switchover to Straight line Method**											
31	Rate	200%										
32	R:	0.4										
33												
34		**DB d(k)**	**SL d(k)**	**max**	**BV(k)**	**Method**	**adjust?**	**check**	**d(k)**			
35	**Year**											**= function**
36	**0**				$5,000							
37	**1**	2000	$1,000	$2,000	$3,000	DB	no	0	$2,000			$2,000
38	**2**	1200	$750	$1,200	$1,800	DB	no	0	$1,200			$1,200
39	**3**	720	$600	$720	$1,080	DB	no	0	$720			$720
40	**4**	432	$540	$540	$540	SL	adj	0	$40			$80
41	**5**	216	$540	$540	$0	SL	adj	1	$40			$0
42												
43												
44	KEY:											
45		= User input										
46		= Unique formula										
47												
48	Unique Formulas											
49	D10	=B3				B32	=B31/B5					
50	B11	=($B$3-$B$4)/$B$5				E36	=B3					
51	C11	=SUM($B$11:B11)				B37	=E36*$B$32					
52	D11	=D10-B11				C37	=E36/($B$5-A37+1)					
53	F11	=SLN($B$3,$B$4,$B$5)				D37	=MAX(B37:C37)					
54						E37	=E36-D37					
55	F21	=B3				F37	=IF(D37=B37,"DB","SL")					
56	B22	=$B$5-A22+1				G37	=IF(E37<$B$4,"ADJ","NO")					
57	C22	=B22/$B$27				H37	=F37="SL"#AND#F36="SL"					
58	D22	=C22*($B$3-$B$4)				I37	=IF(H37=0,J37,I36)					
59	E22	=SUM($D$22:D22)				J37	=IF(G37="ADJ",(E36-$B$4)/($B$5-A37+1),D37)					
60	F22	=F21-D22				L37	=DDB($B$3,$B$4,$B$5,A37)					
61	H22	=SYD($B$3,$B$4,$B$5,A22)										
62	B27	=SUM(B22:B26)										
63												

Template 7.2: MACRS Depreciation Table

Template 7.2 represents a "quick and dirty" approach for determining the MACRS recovery rates. Here, the MACRS table is entered onto the spreadsheet and a table lookup function is used to retrieve values. Note that no calculations are involved.

The table range is outlined with double lines in Template 7.2. The first row in the range contains the class life values (3, 5, 7, 10, 15, and 20). The columns under these values contain the MACRS recovery rates for each year. The year column is not part of the defined table range. Other spreadsheets vary in how the table is defined.

The template contains a single formula in B9 that is copied from B9..B29. This column returns the column from the MACRS table corresponding to the year specified in cell B3.

Two forms of the equation are provided. The formula in row 37 provides values for years 1 through *N*+1, and blank cells for years *N*+2...21, provided global zero suppression is specified on the spreadsheet. If your spreadsheet does not have this feature, use the formula in row 40 to achieve the same results.

	A	B	C	D	E	F	G	H	I	J	K
1	MACRS Depreciation : Look up values in table						Template 7.2				
2											
3	Class Life:	7									
4											
5											
6							MACRS Recovery Rates				
7	Year	r(k)		Year			MACRS Class Life				
8					3	5	7	10	15	20	
9	1	0.1490		1	0.3333	0.2000	0.1490	0.1000	0.0500	0.0375	
10	2	0.2449		2	0.4445	0.3200	0.2449	0.1800	0.0950	0.0722	
11	3	0.1749		3	0.1481	0.1920	0.1749	0.1440	0.0855	0.0668	
12	4	0.1249		4	0.0741	0.1152	0.1249	0.1152	0.0770	0.0618	
13	5	0.0893		5		0.1152	0.0893	0.0922	0.0693	0.0571	
14	6	0.0892		6		0.0576	0.0892	0.0737	0.0623	0.0528	
15	7	0.0893		7			0.0893	0.0655	0.0590	0.0489	
16	8	0.0446		8			0.0446	0.0655	0.0591	0.0452	
17	9	0.0000		9				0.0656	0.0590	0.0447	
18	10	0.0000		10				0.0655	0.0591	0.0447	
19	11	0.0000		11				0.0328	0.0590	0.0446	
20	12	0.0000		12					0.0591	0.0446	
21	13	0.0000		13					0.0590	0.0446	
22	14	0.0000		14					0.0591	0.0446	
23	15	0.0000		15					0.0590	0.0446	
24	16	0.0000		16					0.0295	0.0446	
25	17	0.0000		17						0.0446	
26	18	0.0000		18						0.0446	
27	19	0.0000		19						0.0446	
28	20	0.0000		20						0.0446	
29	21	0.0000		21						0.0223	
30											
31											
32	KEY:										
33		=User Input									
34		=Unique Formula									
35											
36	Unique Formula										
37	B9	=HLOOKUP($B$3,$E$8..$J$29,A9+1)									
38											
39		Alternate Method if Global Zero Suppression is not Available									
40	B9	=IF(HLOOKUP($B$3,$E$8..$J$29,A10)<>0,HLOOKUP($B$3,$E$8..$J$29,A9+1))									
41											

Template 7.3: MACRS Recovery Rate Development

As an alternative to the table lookup approach in the previous template, this spreadsheet derives the MACRS recovery rates from the DB and SL methods. The calculations are based on a cost basis of $1 and a market (salvage) value of $0. Although similar to the declining balance with switch over to straight-line method in Template 7.1, this template is much simpler because the market value is assumed to be zero.

Because of the half-year convention, different formulas are required in years 1, 2-5, and 6. Cell F18 shows that the full basis is depreciated.

The VDB function provides the same results in much less space. Note that it incorporates the half-year convention without additional modifications.

	A	B	C	D	E	F	G	H	I
1	**MACRS Depreciation Percentage Development, Using 1/2 Year Convention**							**Template 7.3**	
2									
3	Declining								
4	Balance %:	200%							
5	Class Life:	5							
6	R:	0.4000							
7									
8		**Remaining**	**Annual**		**Year End**			**@VDB**	
9	**Year**	**Life**	**Depreciation (DB)**	**(SL)**	**BV %**	**r(k)**		**Function**	
10	0				1.0000				
11	1	5.5	0.2000	0.0909	0.8000	0.2000		0.2000	
12	2	4.5	0.3200	0.1778	0.4800	0.3200		0.3200	
13	3	3.5	0.1920	0.1371	0.2880	0.1920		0.1920	
14	4	2.5	0.1152	0.1152	0.1728	0.1152		0.1152	
15	5	1.5	0.0691	0.1152	0.0576	0.1152		0.1152	
16	6	0.5	0.0115	0.0576	0.0000	0.0576		0.0576	
17									
18			Total Depreciation Taken:			1.0000			
19									
20	KEY:								
21		=User Input							
22		=Unique Formula							
23									
24	Unique Formula								
25	B6	+B4/B5							
26	B11	+B5+0.5							
27	C11	+E10*$B$6*0.5							
28	D11	+E10/B11*0.5							
29	E11	=+E10-MAX(C11:D11)							
30	F11	MAX(C11,D11)							
31	C12	=+$B$6*E11							
32	D12	+E11/B12							
33	C16	+$B$6*E15*0.5							
34	D16	+E15/B16*0.5							
35	F18	SUM(F11..F16)							
36									
37	H11	VDB(1,0,$B$5,@MAX(0,A11-1.5),A11-0.5,$B$4,0)							
38									

Template 7.4: ATCF Table

This template is the workhorse for after tax evaluation. It uses the form given in Figure 7-5 in the Tenth Edition of *Engineering Economy* to convert BTCFs to ATCFs.

Cell B8 contains the cost basis, Cells B9: B15 contain the BTCFs, and the market value is in B16. The VDB function is used to determine the MACRS depreciation amounts in column C. Note the negative sign on the first argument in the VDB function. Cell D3, which is hidden, is needed if MACRS depreciation is used. If the class life is greater than or equal to 15, it specifies that 150% declining balance is used in determining r_k, otherwise, 200% declining balance is used.

Any other depreciation method (SL, SYD, etc;) can be used by placing the appropriate formula from the previous templates in Column C. For simplicity, the formula in column C is copied to the end of the class life, which is year 6 for this example. Once this point is reached, no further depreciation is allowed. Template 9.3 provides a more elegant approach than this. The remainder of the table uses the same approach illustrated in the text.

The adjusted ATCF column is necessary because period N (in this example N = 7) Appears twice. The first appearance represents the ordinary BTCFs for that year, while the second represents the consequence of the sale of the asset. This column (G) simply carries forward all ATCFs for years 0 through N -1 and combines the two rows representing year N. This column is used to calculate all financial measures of merit for comparing alternatives. The most common measures of merit appear in cells G18: G20.

	A	B	C	D	E	F	G	H
1	After Tax Analysis, MACRS Depreciation			**Template 7.4**				
2								
3	Tax Rate=	40%		2				
4	MARR =	10%						
5	Class Life:	5						
6	**Year**	**BTCF**	**d(k)**	**Taxable**	**Income**	**ATCF**	**Adjusted**	
7				**Income**	**Tax Due**		**ATCF**	
8	0	($100,000)				($100,000)	($100,000)	
9	1	$20,000	$20,000	$0	$0	$20,000	$20,000	
10	2	$20,000	$32,000	($12,000)	$4,800	$24,800	$24,800	
11	3	$20,000	$19,200	$800	($320)	$19,680	$19,680	
12	4	$20,000	$11,520	$8,480	($3,392)	$16,608	$16,608	
13	5	$20,000	$11,520	$8,480	($3,392)	$16,608	$16,608	
14	6	$20,000	$5,760	$14,240	($5,696)	$14,304	$14,304	
15	7	$20,000	*note*	$20,000	($8,000)	$12,000	$30,000	
16	7	$30,000	*note*	$30,000	($12,000)	$18,000		
17								
18						**NPV =**	($1,411.71)	
19						**AW =**	$290	
20						**IRR =**	9.57%	
21	KEY:							
22		=User input		*note*				
23		=Unique formula		copy depreciation formula only to year (N+1)				
24	D3	If(B5>=15,1.5,2)						
25	F8	B8						
26	G8	F8						
27	C9	VDB(-$B$8,0,$B$5,MAX(0,A9-1.5),A9-0.5,$D$3,0)						
28	D9	B9-C9						
29	E9	-$B$3*D9						
30	F9	B9-E9						
31	G15	F15+F16						
32	G18	NPV(B4,G9:G15)+G8						
33	G19	PMT(B4,7,G18)						
34	G20	IRR(G8:G15,B4)						
35								

Template 7.5:Depletion

Depletion applies to Natural Resources. The calculations are not complex, but the rules regarding percentage depletion can be confusing. This template simplifies the decision process.

Cost depletion applies to all resources. The depletion amount appears in cell C16.

Percentage depletion only applies to certain resources. Cell C22 calculates depletions based on the percentage specified by the IRS. Cell C23 determines 50% of the net income. Percent depletion is the lower of these two values, and the result appears in cell C24.

If a resource qualifies for both depletion methods, the method that gives the largest depletion is selected. The method and amount are identified in cells C27 and D27.

	A	B	C	D	E	F	G	H	I
1	Depletion	**Template 7.5**							
2									
3		Total property purchase	$50,000.00						
4		Cost of land	$10,000.00						
5		Adjusted cost basis	$40,000						
6		IRS Depletion %	10%						
7		Estimated reserves	$400,000						
8									
9		Qt'y removed in year	$130,000.00						
10		Selling price/unit	$0.10						
11		Removal epenses	$11,000.00						
12									
13									
14	**cost depletion**								
15		Depletion unit	$0.10						
16		Total depletion	$13,000.00						
17									
18	**% depletion**								
19		Gross income	$13,000						
20		Less expenses	$11,000						
21		Net income	$2,000						
22		Depletion based on %	$1,300						
23		Maximum depletion allowed	$1,000						
24		% Depletion result	$1,000						
25									
26		If resource qualifies for both depletion methods, select the method that gives the largest depletion	**Method**	**Amount**					
27			Cost Depletion	$13,000					
28									
29									
30									
31	KEY:								
32		User input							
33		Unique formula							
34									
35	Unique Formulas								
36	C15	(B24-B25)/B28							
37	C16	B36*B30							
38	C19	B28*B29							
39	C20	B30							
40	C21	B38-B39							
41	C22	B38*B25							
42	C23	B40*0.5							
43	C24	MIN(B41:B42)							
44	C27	IF(C16>C24,"Cost Depletion", "Percent Depletion")							
45	D27	IF(C16>C24,C16,C24)							

Chapter 8
Developing Cash Flows

Tenth Edition Highlights

Estimating cash flows takes on much greater significance in practice than in the classroom. No model can compensate for unreliable estimates of costs, revenue, or useful life. Ultimately, an organization's success in obtaining business and turning a profit is based on its ability to generate and utilize accurate estimates.

Accuracy is relative, rather than absolute. It is directly proportional to the amount of information available at the time the estimate is made. However, gathering and processing information incurs costs, so more accurate estimates are more expensive. One must ensure that the accuracy of any estimate is matched to its intended use.

Because of the dynamic nature of estimates, they should be formatted to allow for rapid revisions. The work breakdown structure provides a format that is amenable to changes and is well suited for spreadsheet application.

This chapter presents the following models for estimating: cost indexes, factor technique, power sizing technique, and learning curves.

Notation Summary

<u>Cost Indexes</u>

C_k	Price of item in reference year *k*
C_n	Estimated price of item in year *n*
k	Reference year
n	Year for which price is to be estimated
I_n	Index for year *n*
I_k	Index in year *k*

<u>Factor Technique</u>

C	Cost being estimated
C_d	Cost of component *d*, which is estimated directly
f_m	Cost per unit of component *m*
U_m	Number of units of component *m*

<u>Power Sizing Technique</u>

C_A	Cost (Actual dollars) for item A
C_B	Cost (Actual dollars) for item B
S_A	Size of item A
S_B	Size of item B
X	Cost-capacity factor

<u>Learning Curves</u>

K	Input resources required to complete the "first" unit
s	Learning curve slope parameter (decimal)
Z_u	Number of input resource units required to complete unit *u*
u	Output unit number
n	log s / log 2
T_x	Total time to produce *x* units
C_x	Cumulative average time for *x* units

Formula Summary

Cost indexes

$$C_n = C_k\,(I_n/I_k)$$

Factor Technique

$$C = \Sigma C_d + \Sigma f_m U_m$$

Power Sizing

$$C_A = C_B(S_A/S_B)^X$$

Learning curve

$$n = \log(s) / \log(2)$$

$$Z_u = Ku^n$$

$$T_x = K\Sigma u^n$$

$$C_x = T_x/x$$

Overview

The work breakdown structure provides a convenient framework for estimating. The overall project is broken into progressively finer levels of detail. Initially, one may estimate only the highest level components using order of magnitude estimates. As additional information becomes available for each system and subsystem, these values are replaced with more refined estimates. Any of the cost estimating models presented in the chapter can be used in the WBS framework.

The templates for this chapter are straight forward applications of the material from the Tenth Edition of *Engineering Economy* (DeGarmo, Sullivan, Bontadelli, and Wicks). One new function is introduced.

New Features and Functions

ISEMPTY(cell,true,false)

Returns the value specified by the argument *true* if the specified *cell* is empty, otherwise returns the value specified by the *false* argument. Since the space character " " looks the same as a blank cell, it is best to use the delete key to ensure that a cell being tested with this function is really empty.

AVG(range)

Calculates the average of the values specified in *range*.

Spreadsheet Templates

8.1 Work Breakdown Structure
8.2 Estimating Formulas
8.3 Learning Curve

Template 8.1: Work Breakdown Structure

A simple Work Breakdown Structure is illustrated in Template 8.1. The total project cost, cell B3, is the sum of the system costs, cells B5..B7. The System *A* costs are obtained from adding the subsystem costs in cells B10..B11. Systems *B* and *C* are estimated directly. As more information becomes available, these values would be replaced with formulas that refer to the corresponding sub systems.

The sub system costs are obtained by adding the product of the compound quantity and cost. Any change to a quantity or cost value is automatically reflected in every higher level component.

	A	B	C	D	E	F	G	H
1	MACRS Depreciation Percentage Development, Using 1/2 Year Convention							Templa
2								
3	Declining							
4	Balance %:	200%						
5	Class Life:	5						
6	R:	0.4000						
7								
8		**Remaining**	**Annual**		**Year End**			**@VDB**
9	**Year**	**Life**	**Depreciation (DB)**	**(SL)**	**BV %**	**r(k)**		**Function**
10	0				1.0000			
11	1	5.5	0.2000	0.0909	0.8000	0.2000		0.2000
12	2	4.5	0.3200	0.1778	0.4800	0.3200		0.3200
13	3	3.5	0.1920	0.1371	0.2880	0.1920		0.1920
14	4	2.5	0.1152	0.1152	0.1728	0.1152		0.1152
15	5	1.5	0.0691	0.1152	0.0576	0.1152		0.1152
16	6	0.5	0.0115	0.0576	0.0000	0.0576		0.0576
17								
18			Total Depreciation Taken:			1.0000		
19								
20	KEY:							
21		=User Input						
22		=Unique Formula						
23								
24	**Unique Formula**							
25	B6	+B4/B5						
26	B11	+B5+0.5						
27	C11	+E10*$B$6*0.5						
28	D11	+E10/B11*0.5						
29	E11	=+E10-MAX(C11:D11)						
30	F11	MAX(C11,D11)						
31	C12	=+$B$6*E11						
32	D12	+E11/B12						
33	C16	+$B$6*E15*0.5						
34	D16	+E15/B16*0.5						
35	F18	SUM(F11..F16)						
36								
37	H11	VDB(1,0,$B$5, MAX(0,A11-1.5),A11-0.5,$B$4,0)						
38								

Template 8.2: Cost Estimating Models

Template 8.2 contains formulas for Cost Indexes. Power Sizing, and Learning Curves. Each model has n - 1 parameters into the "Given" column and the "Find" column will return the value for the missing parameter. Use the "delete" key to ensure that the "Given" column cell of the parameter that you want to estimate is empty.

The isblank function is introduced to avoid cluttering the "Find" column. If the corresponding "Given" column is empty, the missing parameter is calculated. If the corresponding "Given" column is not empty, the null character is returned, resulting in a blank cell.

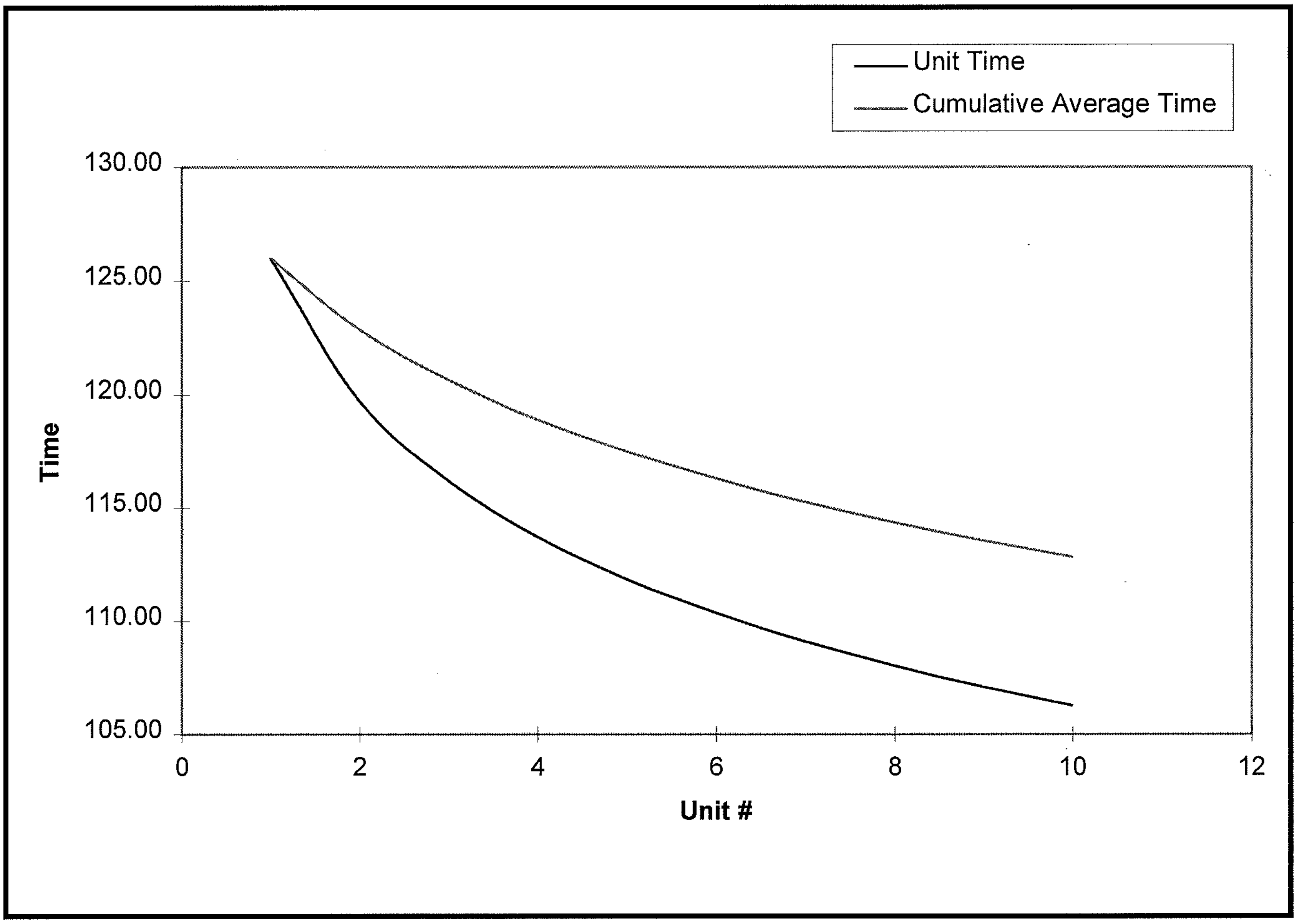
The Backsolver menu command can also be used to accomplish the same results.

	A	B	C	D	E	F	G	H
1	Cost Estimating Models			**Template 8.2**				
2								
3								
4								
5	Cost Index		**GIVEN**	**FIND**				
6		**C(n)**	$350,000					
7		**C(k)**		$525,000				
8		**I(n)**	$312					
9		**I(k)**	$468					
10								
11								
12	Power Sizing		**GIVEN**	**FIND**				
13		**C(A)**		$715				
14		**C(B)**	$300					
15		**S(A)**	$600					
16		**S(B)**	$200					
17		**X**	$1					
18								
19	Learning Curve							
20	**n**	-0.074						
21			**GIVEN**	**FIND**				
22								
23		**K**	126					
24		**s**	0.95					
25		**unit#**	1					
26		**unit time**		126				
27								
28								
29	**KEY:**							
30		=User input						
31		=Unique formula						
32								
33	Unique Formulas							
34	D6	IF(ISBLANK(C6),C7*(C8/C9),"")						
35	D7	IF(ISBLANK(C7),A34*(C9/C8),"")						
36	D8	IF(ISBLANK(C8),C9*(C6/C7),"")						
37	D9	IF(ISBLANK(C9),C8*(C7/C6),"")						
38	D13	IF(ISBLANK($C$13),$C$14*($C$15/$C$16)^$C$17,"")						
39	D14	IF(ISBLANK($C$14),$C$13*($C$15/$C$16)^(-$C$17),"")						
40	D15	IF(ISBLANK($C$15),10^(LOG($C$13/$C$14)/$C$17+LOG(16)),"")						
41	D16	IF(ISBLANK($C$16),10^(-LOG($C$13/$C$14)/$C$17+LOG(15)),"")						
42	D17	IF(ISBLANK($C$17),(LOG($C$13/$C$14)/LOG(15/$C$16)),"")						
43	D23	IF(ISBLANK($C$23),$C$26/($C$25^$B$20),"")						
44	D24	IF(ISBLANK($C$24),10^(LOG(2)*LOG($C$26/$C$23)/LOG($C$25)),"")						
45	D25	IF(ISBLANK($C$25),10^(LOG(2)*LOG($C$26/$C$23)/LOG($C$24)),"")						
46	D26	IF(ISBLANK($C$26),$C$23*$C$25^$B$20,"")						
47								
48								

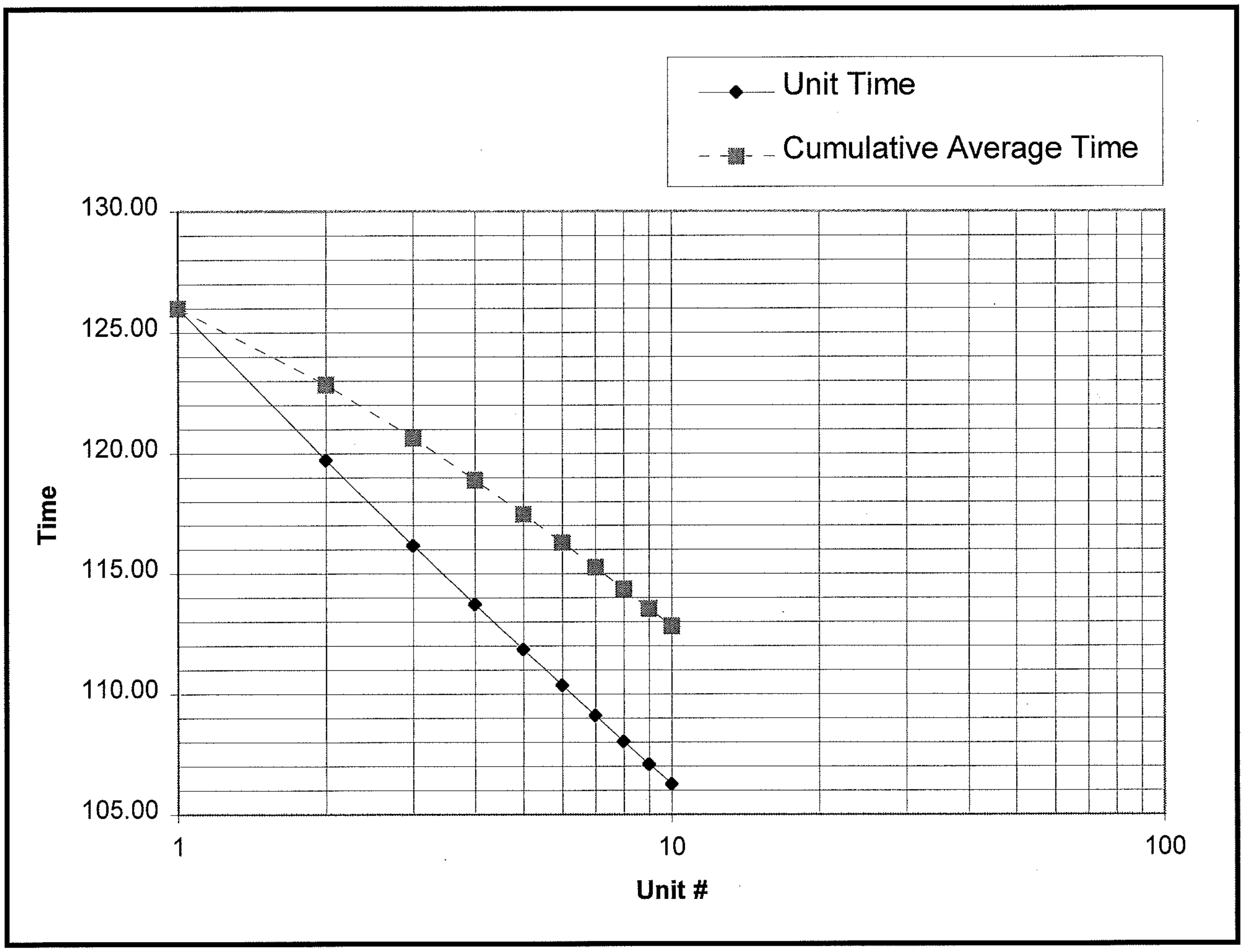
Template 8.3: Learning Curve

Template 8.3 provides unit time, cumulative total time, and cumulative average times based on learning curve theory. The format makes it easy to plot the various statistics versus unit number. The plot of the cumulative average against the unit number is nonlinear. Plotting the log of the unit number against time results in a straight line, which is often preferred when estimating.

	A	B	C	D	E	F	G	H	I
1	Learning Curve				Template 8.3				
2									
3	K =	$126							
4	s =	95.0%							
5	n =	-0.07400							
6									
7	Unit #	Unit Time	Cumulative Total	Cumulative Average					
8	1	126.00	126.00	126.00					
9	2	119.70	245.70	122.85					
10	3	116.16	361.86	120.62					
11	4	113.72	475.58	118.89					
12	5	111.85	587.43	117.49					
13	6	110.35	697.78	116.30					
14	7	109.10	806.89	115.27					
15	8	108.03	914.91	114.36					
16	9	107.09	1022.01	113.56					
17	10	106.26	1128.27	112.83					
18									
19									
20	KEY:								
21		= User input							
22		= Unique formula							
23									
24									
25	Unique Formulas								
26	B5	=LOG(B4)/LOG(2)							
27	B8	=$B$3*A8^$B$5							
28	C8	=SUM($B$8:B8)							
29	D8	=AVERAGE($B$8:B8)							
30									
31									



Unit Time
Cumulative Average Time
130.00
125.00
120.00
115.00
110.00
105.00
Time
0
2
4
6
8
10
12
Unit #


Unit Time
Cumulative Average Time
130.00
125.00
120.00
115.00
110.00
105.00
Time
1
10
100
Unit #

Chapter 9
Inflation and Price Changes

Tenth Edition Highlights

Chapter 9 in the Tenth Edition of *Engineering Economy* (DeGarmo, Sullivan, Bontadelli, and Wicks) presents two additional model refinements: inflation and foreign exchange rates. Although inflation is always present, its magnitude varies over time. During periods of high inflation, it is critical that its effects are incorporated into the analysis. This is particularly true when different commodities escalate at different rates.

Consideration of inflation typically occurs when estimating future cash flows. These future estimates may be either in terms of real dollars or actual dollars, depending on one's view. As long as the appropriate interest rate is used, the PW is the same for both real dollar and actual dollar analyses.

In the real dollar domain, cash flows are expressed in terms of time = 0 purchasing power. The real interest rate considers only the earning power of money. Cash flows in the actual dollar domain are in terms of the current year's purchasing power. The combined interest rate considers both the earning power of money and inflation. To convert between the actual dollar and real dollar domains, one uses factors based on the inflation rate.

Inflation refers to the general increase in the price of goods and services. Some cash flows, such as fixed leases, are immune to inflation. The rate of increase for individual goods and services is often called escalation.

In an increasingly global economy, it is becoming more common for companies to make investments in other countries. Since the currency exchange between the countries varies with time, investment evaluations need to consider this difference. The currency exchange problem is analogous to converting from the real dollar domain to the actual dollar domain. However, the exchange rate, rather than the inflation rate, is used to convert from one currency to another.

Notation Summary

A$	actual dollars
b	base time period
CPI_k	Consumer Price Index for year *k*
e_j'	differential price inflation rate for item *j*
e_j	total price escalation rate
f	general price inflation rate
f_e	annual devaluation rate between U.S. dollar and foreign currency
i_c	combined interest rate
i_{fc}	rate of return relative to foreign country currency
i_{CR}	convenience rate
i_r	real interest rate
i_{US}	rate of return in U.S. dollars
R$	real dollars
$\bar{f}$	average inflation rate

Formula Summary

$$\text{Annual Inflation Rate} = \frac{CPI_k - CPI_{k-1}}{CPI_{k-1}}$$

$$R\$_k = A\$_k \left(\frac{1}{1+f} \right)^{k-b}$$

$$i_r = \frac{i_c - f}{1+f}$$

$$\bar{f} = \left[\prod_{k=1}^{N} (1 + f_k) \right]^{1/N} - 1$$

$$e'_j = \frac{e_j - f}{1+f}$$

$$i_{CR} = \frac{i_c - e_j}{1 + e_j} \quad (A\$ \ Analysis)$$

$$i_{CR} = \frac{i_r - e'_j}{1 + e'_j} \quad (R\$ \ Analysis)$$

$$i_{US} = \frac{i_{fc} - f_e}{1 + f_e}$$

Overview

The calculations required to incorporate the effects of inflation into the analysis are not difficult. However, the relationship between real and actual dollars is often confusing. Once again, spreadsheets can help reduce this confusion.

Template 9.1 takes an annuity expressed in Actual Dollars and converts it to Real Dollars. The template illustrates the size and direction of error in PW if the wrong interest rate is used. The PW is the same in both domains, provided that the correct interest rate is used.

Template 9.2 includes cash flows expressed in Real Dollars and Actual Dollars. Again, if all cash flows are in the same domain and the correct interest rate is used, the PW is the same for both methods.

Template 9.3 is a comprehensive example based on Example 9-8 in the Tenth Edition of *Engineering Economy*. It includes cost components escalating at different rates and determines PW based on the ATCF in both real and actual dollars.

New Features and Functions

ISERR(cell)

Computational errors, such as division by zero, display "ERR" in the cell containing the error and any cells that reference that cell. If the cell contains "ERR," this function returns a logical 1, otherwise it returns a logical 0.

Spreadsheet Templates

9.1	infex	Matching the Interest Rate and Dollar Domain
9.2	infl2	Mixed Domain Analysis
9.3	escalate	Comprehensive Example

Template 9.1: Matching the Interest Rate and the Dollar Domain

Estimates may be expressed in terms of either Real Dollars or Actual Dollars. The present worth is the same for each approach, provided that the correct interest rate is used. All equivalence calculations the Actual Dollar domain should use i_c and all equivalence calculations in the Real Dollar domain should use i_r. To switch between domains, use the inflation rate.

This template shows that both domains yield the same PW if the correct interest rate is used. See cells B20 and C21. It also illustrates the error if the wrong interest rate us used (cells B21 and C20). Real Dollar analysis, coupled with the combined interest rate, understates the PW. The PW is overstated when Actual Dollar analysis is used with the real interest rate. The larger the inflation rate, the larger the error.

	A	B	C	D
1	**Consideration of Inflation**			**Template 9.1**
2	**Errors between Dollars and Rates**			
3				
4	**f =**	6%		
5	**i(c) =**	10.00%		
6	**i(r) =**	3.77%		
7				
8	**Year**	**Actual$**	**Real$**	
9	**1**	$2,000	$1,887	
10	**2**	$2,000	$1,780	
11	**3**	$2,000	$1,679	
12	**4**	$2,000	$1,584	
13	**5**	$2,000	$1,495	
14	**6**	$2,000	$1,410	
15	**7**	$2,000	$1,330	
16	**8**	$2,000	$1,255	
17	**9**	$2,000	$1,184	
18	**10**	$2,000	$1,117	
19				
20	PW i(c)	$12,289	$9,454	
21	PW i(r)	$16,409	$12,291	
22				
23	**KEY:**			
24		User input		
25		Uniqe formula		
26				
27	Unique Formulas			
28	C9	B9/(1+$B$4)^A9		
29	B10	B9		
30	B20	NPV($B$5,B9:B18)		
31	B21	NPV($B$6,B9:B18)		
32				
33				

Template 9.2: Mixed Domain Analysis

Some cash flows may be estimated in Real Dollars, while others are expressed in Actual Dollars. As shown in Template 9.1, which domain to use is not an issue, since the PW is the same for each. It is only necessary that all cash flows be in the same domain. This template illustrates both approaches.

The purchase occurs at time $k = 0$. Thus, cash flows at this time are the same in Actual and Real dollars. The expenses in column C are in Actual Dollars; the savings in Column D are in Real Dollars. Column E provides the conversion factor to switch from one domain to the other, using the inflation rate.

In the Real Dollar analysis, the Actual Dollar expenses are re-expressed in terms of Real Dollars in column F. The expenses and savings are now both in the same terms, and can be added for each year. The net Real Dollar cash flow is used to calculate the PW in cell H17, using the real interest rate.

In the Actual Dollar analysis, the Real Dollar savings are converted to Actual Dollars in column F. The expenses and converted savings can now be added to obtain the net annual cash flow in terms of Actual Dollars. The PW is calculated with cell H28 using the combined interest rate.

In both cases, the PW is the same.

	A	B	C	D	E	F	G	H	I
1	Real$ and Actual$ Comparison				Template 9.2				
2									
3									
4	MARR:	25.00%							
5	f:	8.00%							
6	i (r) :	15.74%							
7									
8	*REAL DOLLAR ANALYSIS*								
9	**Year**	**Purchase**	**A$ Expense**	**R$ Savings**	**A$ -> R$ Conversion**	**R$ Expense**	**Net R$ Cash Flow**		
10	**0**	($50,000)							
11	**1**		($3,000)	$18,000	0.9259	($2,778)	$15,222		
12	**2**		($3,000)	$18,000	0.8573	($2,572)	$15,428		
13	**3**		($3,000)	$18,000	0.7938	($2,381)	$15,619		
14	**4**		($3,000)	$18,000	0.7350	($2,205)	$15,795		
15	**5**		($3,000)	$18,000	0.6806	($2,042)	$15,958		
16									
17							**PW:**	$1,228	
18									
19	*ACTUAL DOLLAR ANALYSIS*								
20	**Year**	**Purchase**	**A$ Expense**	**R$ Savings**	**A$ -> R$ Conversion**	**R$ Expense**	**Net R$ Cash Flow**		
21	**0**	($50,000)							
22	**1**		($3,000)	$18,000	1.0800	$19,440	$16,440		
23	**2**		($3,000)	$18,000	1.1664	$20,995	$17,995		
24	**3**		($3,000)	$18,000	1.2597	$22,675	$19,675		
25	**4**		($3,000)	$18,000	1.3605	$24,489	$21,489		
26	**5**		($3,000)	$18,000	1.4693	$26,448	$23,448		
27									
28							**PW:**	$1,228	
29									
30	KEY:								
31		= User input							
32		= Unique formula							
33									
34	Unique Formulas								
35	B6	=(B4-B5)/(1+B5)							
36	E11	=1+$B$5)^-A11							
37	F11	=C11*E11							
38	G11	=F11+D11							
39	H17	=$B$10+NPV($B$6,$G$11:G15)							
40	E22	=(1+$B$5)^A22							
41	F22	=E22*D22							
42	G22	=F22+C22							
43	H28	=$B$21+NPV($B$4,$G$22:G26)							
44									
45									

Template 9.3: Comprehensive Example

This template is based on Table 9-7, from Example 9-8 in the Tenth Edition of *Engineering Economy.* It illustrates escalation and after tax analysis.

The cash flows and corresponding escalation rates are entered in the upper left region of the spreadsheet. Note that all dollar figures are in terms of 0 dollars. Cell B23 copies the title of the cash flow from the data entry area to the header row of the table. This simple feature provides automatic updates as the cash flow categories change.

Columns B, D, E, F, G, and H use the appropriate escalation rates to express all cash flows in terms of Actual Dollars. These are combined in Column I to obtain the BTCF.

Determining the MACRS depreciation in Column J is a two-step process. Depreciation is actually calculated in Column R with theVDB function. For values of life greater than the MACRS life, the VDB function returns "ERR." Any formulas that refer to these cells will evaluate as " ERR". To avoid this problem, Column J uses the ERR function in combination with an IF function to identify the Column R cells with "ERR". The net result is to copy the MACRS depreciation amounts from Column R to Column J if they are present and to replace the "ERR" with the null character.

Columns K, L, M, and N are identical to the after-tax table in Chapter 7. Column O is the conversion factor necessary to convert from Actual to Real Dollars. Columns P and Q are the Real Dollar ATCF.

Note once again in cells N37 and Q37 that the PW is the same for both the Actual Dollar and the Real Dollar approach.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1	**After Tax Analysis With**			**Template 9.3**				**Unique Formulas**										
2	**Escalation**							C14	=C12+C13-C12*C13				F25	=$B$8*(1+$C$8)^A25				
3		Year 0	Escalation		**KEY:**			C16	=C16+C15+C16*C15				G25	=$B$9*(1+$C$9)^A25				
4		Amount	Rate			=User Input		B23	=A5				H25	=B10				
5	Revenue	$15,000	8.00%			=Unique Formula		C24	=B11				J25	=IF(ISERR(24),,R24)				
6	Materials	($1,200)	10.00%					I24	=SUM(B24:H24)				K25	=I25-J25				
7	Labor	($2,500)	5.50%					M24	=I24+L24				L25	=K25*-$C$14				
8	Energy	($2,500)	15.00%					N24	=M24				H26	=H25				
9	Other Costs	($500)	8.00%					O24	=1/(1+$C$15)^A24				H30	=B10*(1+C10)^5				
10	Leased Equipment	($800)	8.00%					P24	=O24*M24				H31	=H30				
11	Initial Investment	($20,000)						Q24	=P24				N34	=M34+M35				
12	Federal Tax Rate		34.00%					R24	=VDB(-$B$11,0,$B$18,MAX(0,A25-1.5),A25-0.5,2,0)				Q34	=P34+P35				
13	State Tax Rate		7.50%					B25	=$B$5*(1+$C$5)^A25				B35	=-B11*0.1*(1+C15)^10				
14	Effective Tax Rate		38.95%					D25	=$B$6*(1+$C$6)^A25				N37	=NPV(C17,N25:N34)+N24				
15	Inflation Rate		8.00%					E25	=$B$7*(1+$C$7)^A25				Q37	=NPV (C16,Q25:Q34)+P24				
16	i(r)		6%															
17	i(c)		14.48%															
18	MACRS Life	5																
19																		
20																		
21												Cash Flow						
22			Initial				Other	Leased			Taxable	From Income		A$ ATCF	R$		R$ ATCF	MACRS
23	End of Year	Revenue	Investment	Materials	Labor	Energy	Costs	Equipment	A$ BTCF	Deprciation	Income	Taxes	A$ ATCF	Combined	Adjust	R$ ATCF	Combined	Depreciation
24	0		($20,000)						($20,000)				($20,000)	($20,000)	1.00000	($20,000)	($20,000)	$4,000
25	1	$16,200		($1,320)	($2,638)	($2,875)	($540)	($800)	$8,028	$4,000	$4,028	($1,569)	$6,459	$6,459	0.92593	$5,980	$5,980	$6,400
26	2	$17,496		($1,452)	($2,783)	($3,306)	($583)	($800)	$8,572	$6,400	$2,172	($846)	$7,726	$7,726	0.85734	$6,624	$6,624	$3,840
27	3	$18,896		($1,597)	($2,936)	($3,802)	($630)	($800)	$9,131	$3,840	$5,291	($2,061)	$7,070	$7,070	0.79383	$5,612	$5,612	$2,304
28	4	$20,407		($1,757)	($3,097)	($4,373)	($680)	($800)	$9,701	$2,304	$7,397	($2,881)	$6,820	$6,820	0.73503	$5,013	$5,013	$2,304
29	5	$22,040		($1,933)	($3,267)	($5,028)	($735)	($800)	$10,277	$2,304	$7,973	($3,105)	$7,171	$7,171	0.68058	$4,881	$4,881	$1,152
30	6	$23,803		($2,126)	($3,447)	($5,783)	($793)	($1,175)	$10,479	$1,152	$9,327	($3,633)	$6,846	$6,846	0.63017	$4,314	$4,314	
31	7	$25,707		($2,338)	($3,637)	($6,650)	($857)	($1,175)	$11,050		$11,050	($4,304)	$6,746	$6,746	0.58349	$3,936	$3,936	
32	8	$27,764		($2,572)	($3,837)	($7,648)	($925)	($1,175)	$11,606		$11,606	($4,521)	$7,086	$7,086	0.54027	$3,828	$3,828	
33	9	$29,985		($2,830)	($4,048)	($8,795)	($1,000)	($1,175)	$12,138		$12,138	($4,728)	$7,410	$7,410	0.50025	$3,707	$3,707	
34	10	$32,384		($3,112)	($4,270)	($10,114)	($1,079)	($1,175)	$12,632		$12,632	($4,920)	$7,712	$10,348	0.46319	$3,572	$4,793	
35	10	$4,318							$4,318		$4,318	($1,682)	$2,636		0.46319	$1,221		
36																		
37													PW (A$)	$16,798		PW (R$)	$16,798	

Chapter 10
Dealing with Uncertainty

Tenth Edition Highlights

All previous chapters assume that the decision parameters, such as useful life, the MARR, or future revenue, are known with certainty. This assumption is rarely true in practice. Chapter 10 in the Tenth Edition of Engineering Economy (DeGarmo, Sullivan, Bontadelli, and Wicks) provides several methods for incorporating uncertainty into the selection process. Decisions based on these methods are more robust to the inevitable changes that occur than those that do not consider uncertainty.

Decisions under uncertainty refer to the class of problems where probability estimates cannot be assigned to the unknown parameters. Risk analysis, which is covered in Chapter 14, is used when probability estimates are available. The primary methods for handling uncertainty are breakeven analysis, sensitivity analysis, and optimistic-pessimistic estimates.

Breakeven analysis is appropriate when the decision hinges on a single parameter. A measure of merit is calculated for a reasonable range of values of the unknown parameter.

Sensitivity analysis is used when several parameters influence the decision. In its simplest form, each unknown parameter is allowed to vary over a range, with all other parameters held at their most likely values. Plots of the measure of merit against the unknown parameter indicate how influential each parameter is on the decision. It is also possible to allow several parameters to vary simultaneously. Influential parameters may justify the time and expense of gathering additional information to refine the estimates.

Optimistic-pessimistic estimates use three estimates (optimistic, most likely, and pessimistic) for each unknown parameter. The analysis process is similar to that of sensitivity analysis.

Some analysts increase the MARR or decrease the planning horizon for high risk alternatives. These approaches should be avoided, since they also affect the known parameters.

Overview

Templates in this chapter cover breakeven analysis, sensitivity analysis, and optimistic-pessimistic estimates. These are the "what-if" types of analysis responsible for the rapid adoption of electronic spreadsheets in the business community.

The "what-if" table feature is introduced in this chapter. A one-variable table can be used for breakeven analysis. A two-variable table is useful for sensitivity analysis where both variables change simultaneously. Finally, a three-variable table is well suited for optimistic-pessimistic estimates. It can also be used for sensitivity analysis where three variables change simultaneously.

The results of "what-if" tables can be easily duplicated by using formulas with mixed addressing to refer to the uncertain parameters. The formula approach is appealing because changes in the spreadsheet are automatically reflected. With the table approach, one must recalculate the table each time a change is made. Both approaches are illustrated.

New Features

2-variable "what-if" table *Range / Analyze What-if Table ...*

Creates a table of results based on a single formula and ranges of values for two variables.

Enter the range of values for parameter 1 into a column and the range of values for parameter 2 into a row. The first row in the column should be one row below the row for parameter 2. The first column in the row should be one column to the right of parameter 1. The formula goes into the cell where this column and row intersect. Also, designate an input cell for each parameter. These can be any blank cells on the spreadsheet. The formula should refer to the input cells for the values of the two parameters. Enter this information into the dialog box to generate the table.

3-variable "what-if" table *Range / Analyze What-if Table ...*

Creates multiple tables of results based on a single formula and ranges of values for three parameters. These tables appear on consecutive worksheets. Three blank input cells are required. The formula is placed in any cell outside the table ranges and refers to the input cells for values of the three parameters.

Set up a table for two parameters, as described above. Instead of placing the formula in the upper left corner, enter the value of the third parameter into that cell. Copy the table to the same location on consecutive worksheets and enter a different value of the third parameter into each table. There will be one worksheet for each value of the third parameter. When prompted for the table range in the dialog box, enter a 3-d range by highlighting the table on the first worksheet, and then hold the "shift" key and click on the tab for the last worksheet. Complete the information in the dialog box to generate the tables.

Spreadsheet Templates

10.1	bkevn2	Breakeven Analysis, Considering the Time Value of Money
10.2	sensitv1	Sensitivity Analysis, Varying Single Parameter
10.3	sensitv2	Sensitivity Analysis: Varying Two Parameters
10.4	optpess	Optimistic-Pessimistic Estimates

Template 10.1: Breakeven Analysis, Considering the Time Value of Money

Simple breakeven analysis was introduced in Chapter 2. This template enhances the basic model by incorporating the effect of the time value of money.

In this template, incremental revenues and expenses are associated with the purchase of a capital asset. The purchase price, market value, life, and MARR are entered in cells B4..B7. The incremental revenue and expense appear in cells E4..E5. The objective is to identify the breakeven point based on the annual production quantity.

The most direct approach for this class of problem is to determine the equivalent annual worth of the capital expenses. This is simply the capital recovery amount, which is determined in cell B8. The capital recovery is the fixed component of the annual expense. The unit expense is the variable portion of the annual expense.

Columns B and C calculate the annual expense and revenue, based on the number of items produced. The corresponding profit is shown in Column D.

From the profit column, and the graph, we see that the breakeven point is slightly more than 300 units. To estimate the breakeven point more accurately, simply replace the values in Column A with values starting at 300. Alternatively, the solver menu command can be used.

Breakeven problems can also be solved using the 1-variable "what-if" table. However, for a single variable, the approach presented here is usually more convenient.

Template 10.2: Sensitivity Analysis, Varying Single Parameter

A primary spreadsheet application is to answer "what-if" types of questions. This template examines the impact of changes in the interest rate (MARR), useful life, annual savings, and market value on the PW of an investment. Since more than one parameter may vary, sensitivity analysis is appropriate.

One approach to sensitivity analysis is to hold all parameters at their nominal values and vary a single parameter over its range. A second parameter is then selected to vary over its range while all others are held at their nominal values. The process is repeated until all parameters are evaluated. This is the approach illustrated in this template, with each parameter varying over a range of +/- 50% from nominal.

The nominal values for each parameter appear in cells B4..B8. The percent change from nominal for each parameter is in Column A. Columns B through E each contain a unique formula that refers to the nominal values to determine PW. The particular parameter of interest, for example the MARR (*i*) in column B, is multiplied by the factor (1 + % change) to create the table. If the formulas are correct, the PW in each column will be equal at the nominal value (% change = 0).

Graphing the PW results simplifies the task of identifying the critical parameters. The percent change column contains the values for the independent axis and columns B through E contain the values for the dependent variables.

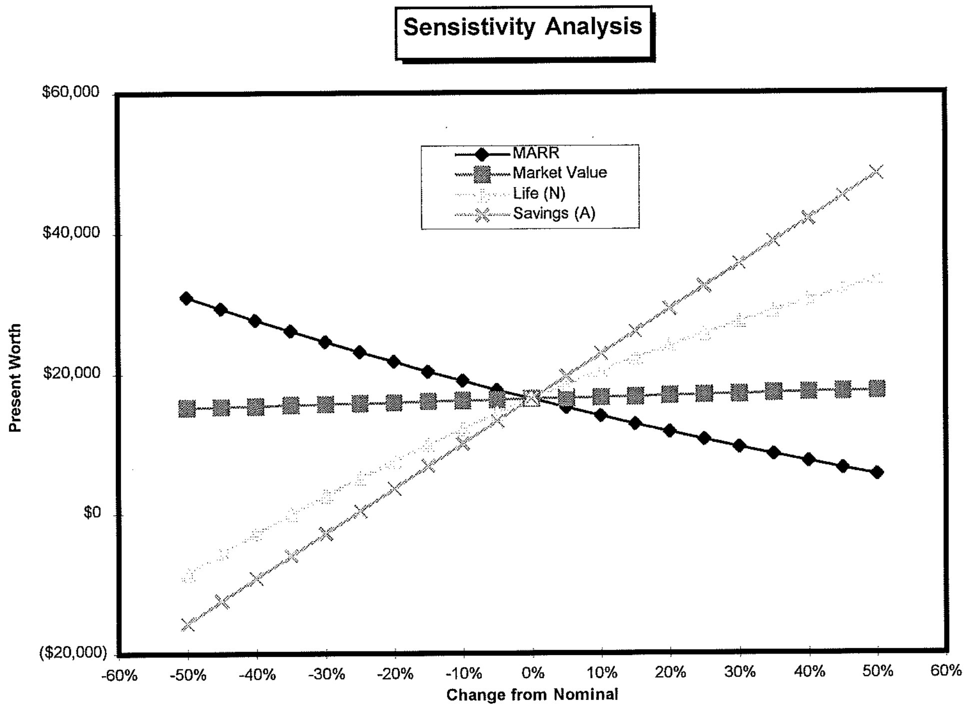
Since *annual savings* has the steepest slope, PW is most sensitive to the annual savings. The next most influential parameter is *useful life*. Note that these are the only two parameters that can result in a negative PW, for the range of values considered. Higher values of *annual savings* and *useful life* increase the PW.

The least sensitive parameter is *market value*. It can vary over a large range with little effect on PW. This is as expected, since the market value is low and occurs at the end of the study period. Larger values of the MARR decrease the PW, as we learned in Chapter 3, but never cause it to become negative in the range considered in this example. Armed with this information, the analyst would leave the estimate of the market value as it is and refine the estimates for the annual savings and useful life. Note that the annual savings and market value curves are linear, while the MARR and life curves are nonlinear.

Template 10.2: Sensitivity Analysis, Varying Single

A primary spreadsheet application is to answer "what-if" types of questions. This template examines the impact of changes in the interest rate (MARR), useful life, annual savings, and market value on the PW of an investment. Since more than one parameter may vary, sensitivity analysis is appropriate.

	A	B	C	D	E	F	G	H	I	J
1	Sensitivity Analysis			KEY:		Template 10.2				
2					= User input					
3	Nominal Values				= Unique formula					
4	Purchase:	($50,000)								
5	MARR:	10%		Unique Formulas						
6	Market Value:	$5,000		B12	=$B$4+(PV($B$5*(1+A12),$B$7,$B$8)*-1)+$B$6/((1+$B$5*(1+A12))^$B$7)					
7	Life (N):	8		C12	=$B$4+(PV($B$5,$B$7,$B$8)*-1)+($B$6*(1+A12))/(1+$B$5)^$B$7					
8	Savings (A):	$12,000		D12	=$B$4+(PV($B$5,$B$7*(1+A12),$B$8)*-1)+$B$6/(1+$B$5)^($B$7*(1+A12))					
9				E12	=$B$4+(PV($B$5,$B$7,$B$8*(1+A12))*-1)+$B$6/(1+$B$5)^$B$7					
10										
11	**% Change**	**MARR**	**Market Value**	**Life (N)**	**Savings (A)**					
12	-50%	$30,943	$15,185	($8,547)	($15,658)					
13	-45%	$29,273	$15,302	($5,608)	($12,457)					
14	-40%	$27,655	$15,419	($2,780)	($9,256)					
15	-35%	$26,086	$15,535	($58)	($6,055)					
16	-30%	$24,566	$15,652	$2,563	($2,854)					
17	-25%	$23,091	$15,769	$5,085	$347					
18	-20%	$21,661	$15,885	$7,514	$3,548					
19	-15%	$20,274	$16,002	$9,851	$6,749					
20	-10%	$18,927	$16,118	$12,101	$9,950					
21	-5%	$17,620	$16,235	$14,267	$13,151					
22	0%	$16,352	$16,352	$16,352	$16,352					
23	5%	$15,120	$16,468	$18,358	$19,553					
24	10%	$13,923	$16,585	$20,290	$22,754					
25	15%	$12,761	$16,702	$22,150	$25,955					
26	20%	$11,631	$16,818	$23,940	$29,155					
27	25%	$10,533	$16,935	$25,663	$32,356					
28	30%	$9,466	$17,051	$27,321	$35,557					
29	35%	$8,428	$17,168	$28,918	$38,758					
30	40%	$7,419	$17,285	$30,454	$41,959					
31	45%	$6,437	$17,401	$31,934	$45,160					
32	50%	$5,482	$17,518	$33,357	$48,361					
33										
34										
35										


Sensistivity Analysis
MARR
Market Value
Life (N)
Savings (A)
$60,000
$40,000
$20,000
$0
($20,000)
Present Worth
-60%
-50%
-40%
-30%
-20%
-10%
0%
10%
20%
30%
40%
50%
60%
Change from Nominal

Template 10.3: Sensitivity Analysis: Varying Two Parameters

An alternative approach to sensitivity analysis is to allow the model parameters to take on different values simultaneously. This more closely represents what happens in practice.

Template 10.3 presents two methods for two parameter sensitivity analysis. One is based on the "what-if" table command; the other uses a formula with mixed addressing. The same data from Template 10.2 is considered. Here the *MARR* and *market value* are fixed, while the *life* and *annual savings* parameters vary simultaneously.

For the tabular method, select two empty cells for input cells one and two. Next, create a column of values (B16..B36) over which the first parameter will vary. One row up and one cell to the right, create a row of values (C15..M15) for the second parameter. Note that the increment for the row is different from the increment for the column. Place the formula to compute PW where this column and row intersect (B15). The formula should refer to the input cell locations for values of the parameters that will vary simultaneously.

Access the "what-if" table menu command dialog box. Specify 2 variables, the address for the input cells (B10 and B11), and the table range (B15..M36). After clicking "OK," the spreadsheet will compute the PW for each combination of annual savings and life and fill in the table.

An alternative method is to create the row and column for the annual savings and life values and place a different formula into cell C16. This formula uses mixed addressing to refer to the base row and base column. The formula is copied to fill the table.

Although both approaches give the same results, many analysts prefer the formula approach. Changes to any of the user input cells will be automatically reflected in the table of PW values. With the "what-if" table menu command approach, one must access the dialog box to update the table.

One can scan the completed table to identify which combination of values for annual savings and life results in negative PW values. Selecting the "display negative values in red" option makes undesirable combinations stand out. The PW values in each cell represent points on a surface above the savings/life grid. Figure 10.1 is a 3-d plot of the surface done with *Excel*.

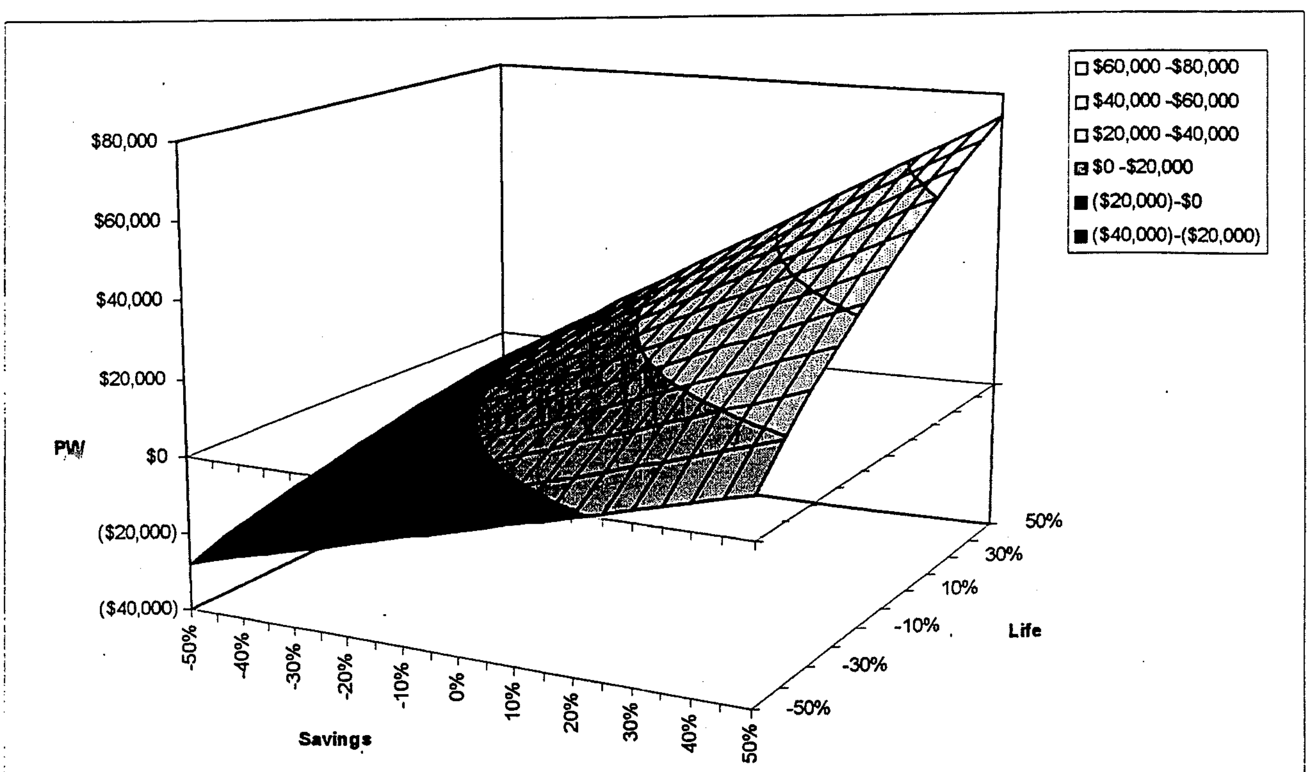


Figure 10.1. Surface plot of PW from two factor sensitivity table.

	A	B	C	D	E	F	G	H	I	J	K	L
1	**Two Factor Sensitivity Analysis**			**KEY:**			**Template 10.3**					
2					=User Input							
3	**Nominal Values**				=Unique Formula							
4	Purchase:	($50,000)			=Table Input Cells							
5	MARR:	10%										
6	Market Value:	$5,000		**Unique Formula: Data Table**								
7	Life (N)	8		B15	=B4+PV(B5,B7*(1+B10),-B8*(1+B11))+B6/(1+B5)^(B7*(1+B10))							
8	Savings (A)	$12,000										
9				**Unique Formula: Mixed Addressing**								
10	**Life %**			C16	=$B$4+PV($B$5,$B$7*(1+C$15),-$B$8*(1+$B16))+$B$6/(1+$B$5)^($B$7*(1+C$15))							
11	**Savings %**											
12												
13												
14								**LIFE**				
15		**$16,351.65**	**-50%**	**-40%**	**-30%**	**-20%**	**-10%**	**0%**	**10%**	**20%**	**30%**	**40%**
16		**-50%**	($27,566)	($24,808)	($22,253)	($19,885)	($17,691)	($15,658)	($13,774)	($12,029)	($10,412)	($8,913)
17		**-45%**	($25,664)	($22,605)	($19,771)	($17,145)	($14,712)	($12,457)	($10,368)	($8,432)	($6,638)	($4,976)
18		**-40%**	($23,762)	($20,402)	($17,289)	($14,405)	($11,732)	($9,256)	($6,961)	($4,835)	($2,865)	($1,040)
19		**-35%**	($21,860)	($18,200)	($14,808)	($11,665)	($8,753)	($6,055)	($3,555)	($1,238)	$908	$2,897
20		**-30%**	($19,958)	($15,997)	($12,326)	($8,925)	($5,774)	($2,854)	($148)	$2,358	$4,681	$6,834
21		**-25%**	($18,056)	($13,794)	($9,845)	($6,186)	($2,795)	$347	$3,258	$5,955	$8,455	$10,771
22		**-20%**	($16,154)	($11,591)	($7,363)	($3,446)	$184	$3,548	$6,664	$9,552	$12,228	$14,707
23		**-15%**	($14,252)	($9,388)	($4,882)	($706)	$3,164	$6,749	$10,071	$13,149	$16,001	$18,644
24		**-10%**	($12,350)	($7,186)	($2,400)	$2,034	$6,143	$9,950	$13,477	$16,746	$19,774	$22,581
25		**-5%**	($10,448)	($4,983)	$81	$4,774	$9,122	$13,151	$16,884	$20,343	$23,548	$26,518
26	**ANNUAL**	**0%**	($8,547)	($2,780)	$2,563	$7,514	$12,101	$16,352	$20,290	$23,940	$27,321	$30,454
27	**SAVINGS**	**5%**	($6,645)	($577)	$5,044	$10,254	$15,080	$19,553	$23,697	$27,536	$31,094	$34,391
28		**10%**	($4,743)	$1,625	$7,526	$12,993	$18,059	$22,754	$27,103	$31,133	$34,868	$38,328
29		**15%**	($2,841)	$3,828	$10,008	$15,733	$21,039	$25,955	$30,510	$34,730	$38,641	$42,264
30		**20%**	($939)	$6,031	$12,489	$18,473	$24,018	$29,155	$33,916	$38,327	$42,414	$46,201
31		**25%**	$963	$8,234	$14,971	$21,213	$26,997	$32,356	$37,322	$41,924	$46,187	$50,138
32		**30%**	$2,865	$10,436	$17,452	$23,953	$29,976	$35,557	$40,729	$45,521	$49,961	$54,075
33		**35%**	$4,767	$12,639	$19,934	$26,693	$32,955	$38,758	$44,135	$49,117	$53,734	$58,011
34		**40%**	$6,669	$14,842	$22,415	$29,432	$35,935	$41,959	$47,542	$52,714	$57,507	$61,948
35		**45%**	$8,571	$17,045	$24,897	$32,172	$38,914	$45,160	$50,948	$56,311	$61,280	$65,885
36		**50%**	$10,473	$19,248	$27,378	$34,912	$41,893	$48,361	$54,355	$59,908	$65,054	$69,822
37												

Template 10.4: Optimistic- Pessimistic Estimates

Instead of allowing variables to take on a large number of values, the optimistic-pessimistic estimate approach is based on three estimates: optimistic, most likely, and pessimistic. This is a special case of three parameter sensitivity analysis.

This template is based on Table 10-4 in the Tenth Edition of *Engineering Economy*. As in Template 10.3, one may use the "what-if " table menu command approach or the formula approach. The table menu command approach requires multiple worksheets, a feature which is not available on older spreadsheet versions.

Begin by setting aside three blank input cells and placing the formula outside of any table ranges. The formula should refer to the input cells for the value of the parameters. Note that the formula in C11 returns " ERR " since the input cells are blank. This does not affect the spreadsheet.

On the first worksheet, create a table with values of parameters 1 in a column on the left side and values of parameter 2 in a row across the top. At the intersection of the row and column, enter the value for parameter 3. This is the same setup as the two variable case in Template 10.3, except that the third variable replaces the formula. Copy this table to consecutive worksheets, making sure that it is in the same location. Enter a different value for the third parameter on each worksheet.

From the " what-if " table menu command dialog box, specify "3 variables," the address of each input cell, and the address of the formula. Enter the range for the table on the first worksheet, and make it a three-dimensional range by holding the " shift " key while clicking on the tab of the last worksheet. Finally, click " OK " to calculate the values in all tables. For convenience, all tables were copied to the first worksheet in this template.

Alternatively, one can create the three tables as above on a single worksheet. Enter a formula into the upper left corner of each that uses mixed addressing to refer to values of each parameter.

As in Template 10.3, the results are the same for both methods. The formula approach offers the advantage of automatically updating the tables whenever changes are made to the spreadsheet.

	A	B	C	D	E	F
1	**Optimistic Pessimistic Estimates**				**Template 10.4**	
2						
3	Investment	$150,000				
4	Market Value	$0				
5	Marr	8.00%				
6						
7	**Cell Numbe**	**Variable Name**	**Input Cells**			
8	1	Annual Revenue				
9	2	Annual Expense				
10	3	Life				
11		Formula	ERR			
12						
13						
14	Sheet1	$20,000	18	10	8	
15		$110,000	($16,005)	($22,354)	($26,102)	
16		$70,000	($16,005)	($22,354)	($26,102)	
17		$50,000	($16,005)	($22,354)	($26,102)	
18						
19	Sheet2	$43,000	18	10	8	
20		$110,000	($16,005)	($22,354)	($26,102)	
21		$70,000	($16,005)	($22,354)	($26,102)	
22		$50,000	($16,005)	($22,354)	($26,102)	
23						
24	Sheet3	$57,000	18	10	8	
25		$110,000	($16,005)	($22,354)	($26,102)	
26		$70,000	($16,005)	($22,354)	($26,102)	
27		$50,000	($16,005)	($22,354)	($26,102)	
28						
29						
30						
31	KEY:					
32		=User Input				
33		=Unique Formula				
34		=Table Input Cell				
35						
36	Unique Formula: Data Table for 3 Variables					
37	C11	=PMT(-B3,B5,C9)+B4/(1+B5)^C9+C8-C10				
38						
39	Unique Formulas: Mixed Addressing					
40	C15	=PMT($B$5,C$14,$B$3)+$B$4/(1+$B$5)^C$14+$G15-$G$14				
41	C20	=PMT($B$5,C$19,$B$3)+$B$4/(1+$B$5)^C$14+$G15-$G$19				
42	C25	=PMT($B$5,C$24,$B$3)+$B$4/(1+$B$5)^C$14+$G15-$G$24				

Template 11.1: Before-tax Economic Life

This template determines the before-tax economic life of an asset. The data are from Example 11-3 in the Tenth Edition of *Engineering Economy.* The final results are the same as Table 11-4, but the column headings differ. This is because the template uses the capital recovery approach suggested in the example.

The capital recovery formula in column D uses absolute addressing for the purchase price and relative addressing to obtain the current year market value. Column F calculates the PW of the values in column E. Column G determines the EUAC of the yearly expenses. The cumulative EUAC column (H) is the sum of columns D and G. The IF function in column I places a label that identifies the minimum EUAC value, while the IF function in column A identifies the corresponding economic life.

	A	B	C	D	E	F	G	H	I
1	Economic Life, Before Tax				Template 11.1				
2									
3	MARR =	10.00%							
4									
5			End of	CR	Annual	PW of	EUAC of	Cumulative	
6		Year	Year MV	Amount	Expense	Annual Exp.	Ann. Exp.	EUAC	
7		0	$20,000						
8		1	$15,000	**$7,000**	$2,000	**$1,818**	**$2,000**	**$9,000**	
9		2	$11,250	$6,167	$3,000	$2,479	$2,476	$8,643	
10	Econ. Life->	3	$8,500	$5,474	$4,620	$3,471	$3,124	$8,598	<-MIN EUAC
11		4	$6,500	$4,909	$8,000	$5,464	$4,175	$9,083	
12		5	$4,750	$4,498	$12,000	$7,451	$5,456	$9,954	
13									
14									
15	KEY:								
16		=User Input							
17		=Unique Formula							
18									
19	Unique Formulas								
20	A8	=IF(H8=MIN($H$8..$H$12),"Econ. Life->","")							
21	D8	=PMT($B$3,B8,-($C$7-C8))+C8*$B$3							
22	F8	=E8/(1+$B$3)^B8							
23	G8	=PMT($B$3,B8,-SUM($F$8...F8))							
24	H8	=G8+D8							
25	I8	=IF(H8=MIN($H$8..$H$12),"<-MIN EUAC","")							
26									