Orbit Odyssey

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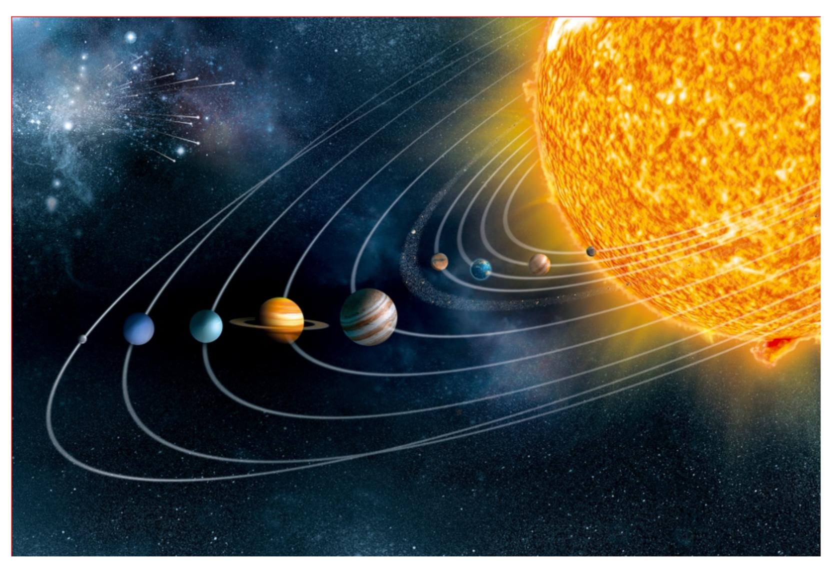
Introduction

In the past, it was believed that Earth was at the center of our solar system, but new discoveries have since proven that to be untrue. As time went on, scientists discovered new planets and realized that the positions of the planets were not as they had originally calculated, due to the influence of other forces. Planetary motion is the motion of planets and other celestial bodies as they are subjected to forces that create conic section orbits. In my work, I will be using a system of first-order differential equations to numerically solve and graph the orbits of the four inner planets.



Mathematical Model

Force

Force is the gravitational attraction between two objects. .

$$\vec{F}(x_j, x_k) = \frac{G m_j m_k}{\|x_j - x_k\|^2} * \frac{x_j - x_k}{\|x_j - x_k\|}$$

Node Of Planets

$$\mathrm{m}_j \ddot{x} = \textstyle\sum_{k=1}^{n} F_j(x_j, x_k)$$

$$\mathrm{m}_{Sun} \ddot{x}_{Sun} = F_{Mercury} + F_{Venus} + F_{Earth} + F_{Mars}$$

$$\mathrm{m}_{Merc} \ddot{x}_{Mercury} = F_{Sun} + F_{Venus} + F_{Earth} + F_{Mars}$$

$$\mathrm{m}_{Venus} \ddot{x}_{Venus} = F_{Sun} + F_{Mercury} + F_{Earth} + F_{Mars}$$

$$\mathrm{m}_{Earth} \ddot{x}_{Earth} = F_{Sun} + F_{Mercury} + F_{Venus} + F_{Mars}$$

$$\mathrm{m}_{Mars} \ddot{x}_{Mars} = F_{Sun} + F_{Mercury} + F_{Venus} + F_{Earth}$$

Numerical Methods

Euler Explicit

The Euler method involves taking small steps along the slope of the ODE at each point to estimate the solution at the next point

$$y_{j+1} = y_j + h f(t_j, y_j)$$

Runge-Kutta Method

The Runge-Kutta method involves estimating the value of the solution at a series of intermediate points and using these estimates to improve the accuracy of the approximation.

$$k_1 = f(t_j, y_j)$$

$$k_2 = f(t_j + \frac{1}{2}h_j, y_j + \frac{1}{2}h_j k_1)$$

$$k_3 = f(t_j + \frac{1}{2}h_j, y_j + \frac{1}{2}h_j k_2)$$

$$k_4 = f(t_j + h_j, y_j + h_j k_3)$$

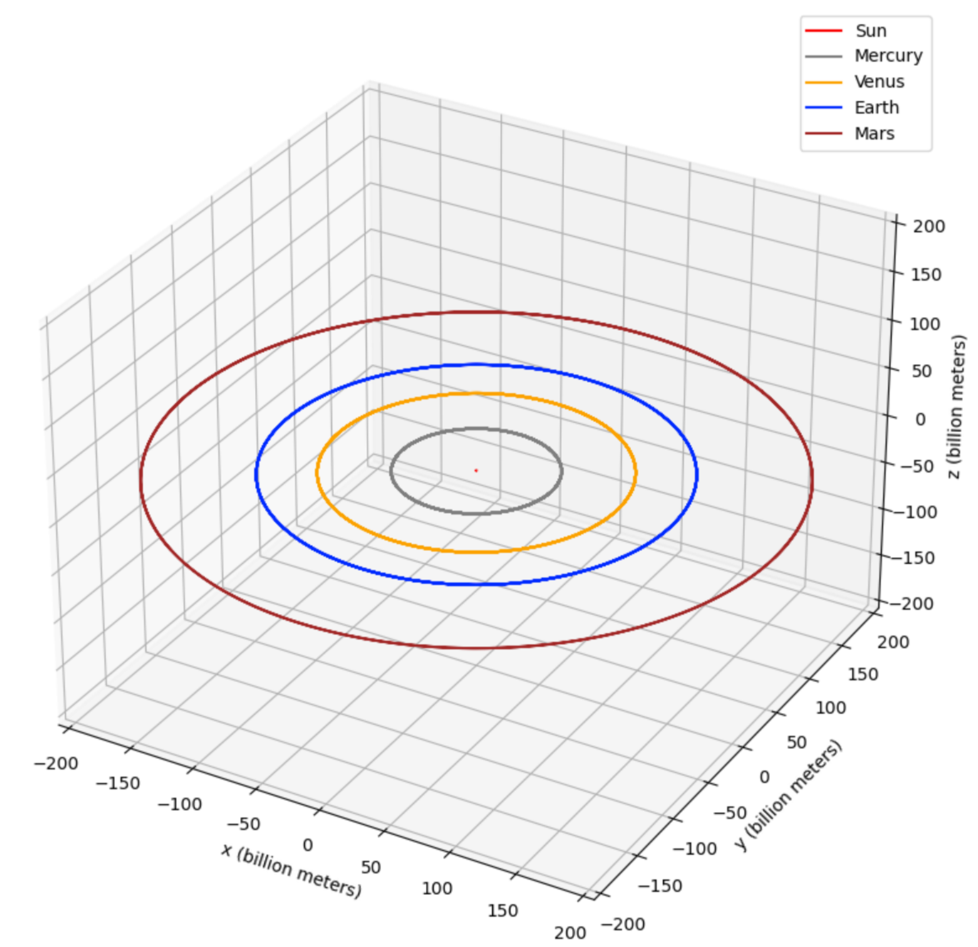
$$y_{j+1} = y_i + h_j\left((k_1/6) + (k_2/3) + (k_3/3) + (k_4/6)\right)$$

Implementing Numerical Methods

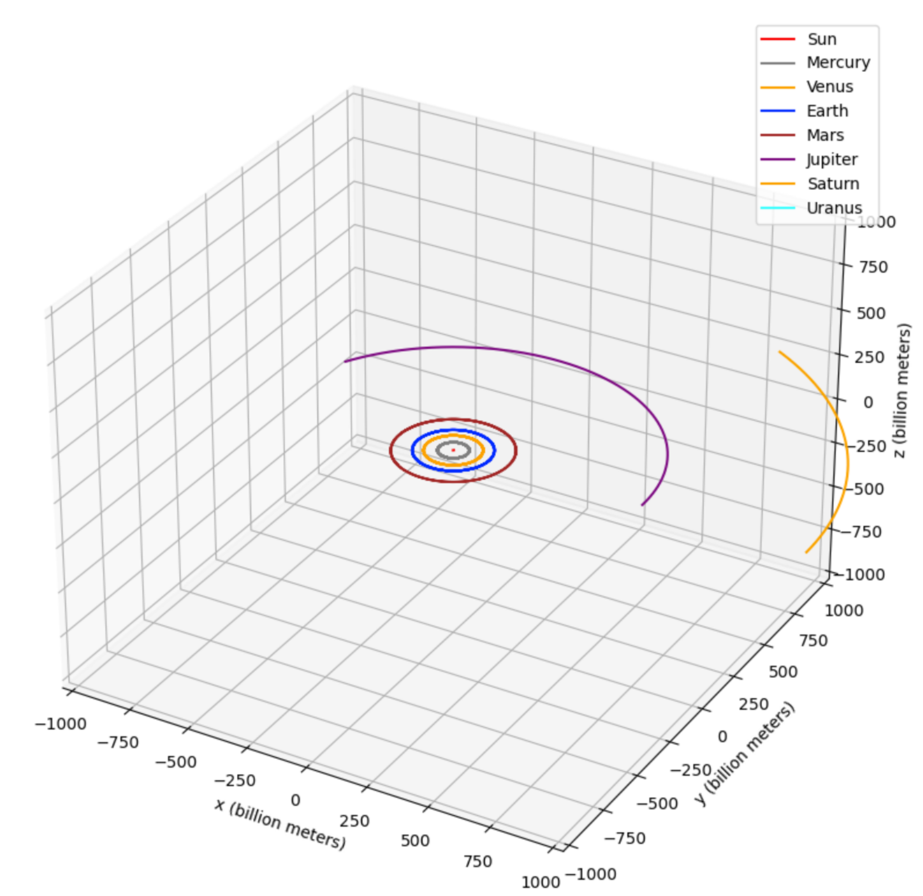
```python
# Apply the Runge-Kutta method
for i in range(len(time) - 1):
    k1 = dt * f(time[i], state)
    k2 = dt * f(time[i] + dt / 2, state + k1 / 2)
    k3 = dt * f(time[i] + dt / 2, state + k2 / 2)
    k4 = dt * f(time[i] + dt, state + k3)
    state = state + (1/6) * (k1 + 2 * k2 + 2 * k3 + k4)
    solution[i+1] = state.copy()
```

The Runge-Kutta method is an effective numerical method for solving the equations of motion for the planets in our solar system. By adjusting the time step and duration of the simulation, we can obtain a more accurate and detailed representation of the motion of the planets.

Python Code Simulation



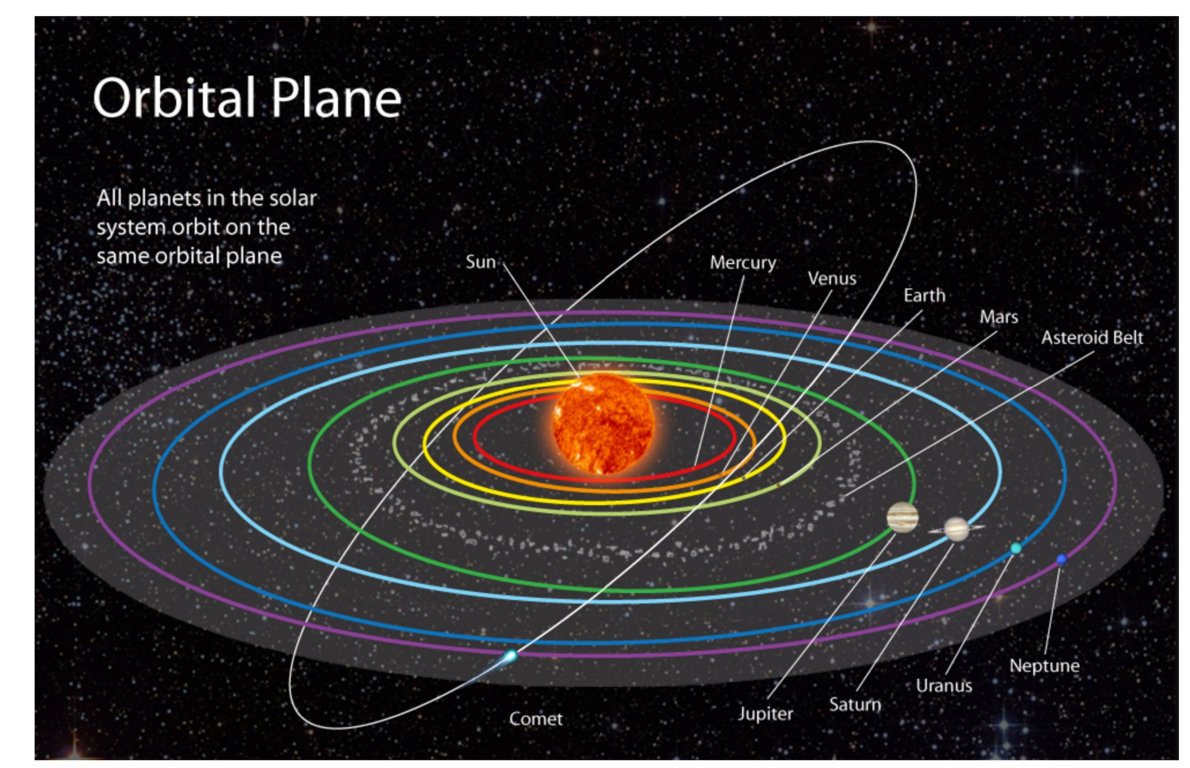
Elliptical orbits produced by the four inner planets.



Orbits length of the outer planets: Jupiter, Saturn, Uranus.

Conclusion

This project has demonstrated the power and potential of using numerical simulations to model complex systems and phenomena, such as the motion of planets in our solar system. By implementing the Runge-Kutta method to solve the equations of motion, we were able to accurately simulate the positions and velocities of the planets over time. It was fun and engaging to learn about programming, numerical methods, and scientific modeling. I took inspiration after doing a similar assignment in class, however, I wanted mine to be accurate to our solar system. By experimenting with the code I was able to see how changes in parameters or initial conditions affect the motion of the planets and gain a deeper appreciation for the complexities of our universe.



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