

Applied Mathematics Seminar

Friday, March 25, 2pm, DON 2159

Donuts, pretzels, fluid flow, and ranking sports teams: a short tour of discretizing Laplacians on meshes and graphs.

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Exterior calculus is a language to express the equations of elasticity, electromagnetism and fluid flow that is finding increasing acceptance. In this talk, we shall first provide a brief overview of exterior calculus and how it generalizes vector calculus. We shall then introduce the notion of Hodge decomposition which is a central feature in this framework and generalizes the well known Helmholtz decomposition in vector calculus. Next, we shall connect the Hodge decomposition to analysis (in particular, the Laplacian operator), and topology (roughly speaking, what is the shape of a space? Think donuts, coffee mugs and pretzels!). In this backdrop, we shall discuss a discrete method for computing harmonics (the kernel of the Laplacian). All of this would set us up to discuss solutions of Poisson partial differential equations (PDEs) using two different discretizations of exterior calculus: a finite element approach and a geometric one. We shall provide some computational examples of how these seemingly different methods are related. Finally, if we have any time left, we shall show how these ideas can model PDEs on graphs that can be suitably interpreted as ranking pairwise comparison data, for example, sports teams.

Students are very welcome to the Informal Discussion Seminar. We assume knowledge in mathematics and physics at the level of Calculus III and Physics II.